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Optimal Consensus of Euler-Lagrangian Systems with Kinematic Constraints

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Abstract: In this paper, we study a distributed constrained consensus problem for heterogeneous Euler-Lagrange (EL) systems with kinematic constraints. Each agent is assigned with a convex function as individual cost, and we design a distributed control law to achieve consensus at the optimum of the aggregate cost under given constraints on the velocity and control input which are required to be bounded. Noticing that an EL system with exact knowledge of nonlinearities can be turned into a double-integrator, we first explore the consensus of double-integrator multi-agent systems by Lyapunov method, then extend the result to the case of EL dynamics by inverse dynamics control. Specifically, with knowledge of the furthest distance from the optimum to initial positions, it is shown that control gains can be properly selected to achieve an exponentially fast convergence while satisfying the bounded kinematic constraints, if the fixed undirected topology is connected. A numeric example is given to illustrate the result.

1. INTRODUCTION

As a basic problem in cooperative control of multi-agent systems, distributed consensus aims to achieve an agreement on specific quantities among different agents via exchange of local information , e.g. a coordinated movement in flocking and formation, a common estimate of some global parameter in sensor fusion and tracking. Over recent years, distributed consensus method has also been combined with optimization techniques to solve the optimal consensus problem, where the final consensus value is required to minimize a global cost function. The optimal consensus problem was firstly addressed as a distributed optimization problem arising from parallel and distributed computing, which seeks the optimum of the aggregate cost as the sum of individual costs assigned to each computer node and finds application in areas such as resource allocation and distributed learning. The individual cost is usually taken as a convex function, and subgradient-based methods are commonly employed to solve the primal problem (e.g. Nedic and Ozdaglar (2009); Nedic et al. (2010)), or the Lagrangian dual problem if there are additional equality or inequality constraints (e.g. Yuan et al. (2011); Minghui Zhu and Martinez (2012)). Note that in these works the algorithm is implemented in discrete-time iteration, which does not involve the dynamics of each node.

For practical physical systems such as robots and unmanned vehicles, it is intriguing and promising to implement distributed optimization methods to improve the performance of collective tasks. In this case specific continuous-time dynamics has to be considered in the modeling of each node and nonlinear control techniques are applied. Wang and Elia (2011) and Kia et al. (2015) dealt with the optimal consensus problem in the case of differentiable individual costs by including an integral feedback of consensus error, which avoids the issue of using a diminishing gain on the gradient term and is robust to disturbance. Qiu et al. (2016) solved the optimal consensus problem with a common set constraint by using subgradients and projection, while Liu and Wang (2015) solved a similar problem for double-integrator multi-agent networks. Some other works focused on higher-order dynamics. Zhang and Hong (2014) investigated the optimal consensus of double-integrator multi-agent systems, while Deng and Hong (2016) achieved a semi-global convergence when the dynamics of each agent is described by Euler-Lagrange (EL) equations. Note that the above works did not consider the constraints on actuators, which may deter the implementation on physical systems.

In this paper, we study the distributed optimal consensus problem of multi-agent systems with EL dynamics under given kinematic constraints. EL equation has been used for modeling a wide class of mechanical systems including robots and spacecrafts. When fulfilling cooperative tasks with unmanned systems such as Unmanned Aerial Vehicles (UAVs), certain constraints on the velocity and acceleration have to be taken into account due to safety and actuator limits. To formulate the problem, we assign each agent with a convex function as its individual cost, and aim to design a distributed control law to achieve consensus at the optimum of the aggregate cost under bounded velocity and control input constraints.

Noticing that any EL system with exact knowledge of nonlinearities can be transformed into a double-integrator, we first explore the optimal consensus under kinematic constraints for double-integrator multi-agent systems. In this case, the control input of each agent consists of a

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weighted sum of position differences from its neighbors, a damping term of its velocity, a gradient descent of its individual cost, as well as an integral feedback of the consensus error to correct the gradient differences. With the aid of a quadratic Lyapunov function, we show that with the designed control laws the state of each agent converges exponentially fast to the global optimum under a fixed and connected topology. Furthermore, if the upper bound of the maximum distance between the optimum and initial positions can be obtained, then the control gains can be properly chosen to satisfy kinematic constraints. Once the convergence of double-integrator systems is established, the convergence of the corresponding EL systems follows naturally by employing the inverse dynamics control, or the computed torque. It should be mentioned that the present work is built upon the work of Zhang and Hong (2014) by revising the Lyapunov function therein and tuning the control gains to achieve an exponential stability under constraints, while Zhang and Hong (2014) only achieved an asymptotic stability without considering constraints.

The rest of this paper is organized as follows. Some preliminaries about graph theory and convex functions are briefly reviewed in Section 2, and we formulate the problem under investigation in Section 3. After establishing the optimal consensus for double-integrator multi-agent systems in Section 4.1, we proceed to the case of EL dynamics under kinematic constraints in Section 4.2. A numeric example is provided to illustrate the result in Section 5, and the whole work is concluded in Section 6.

Notations: A vector x is always viewed as a column vector, and $\mathbf{1}$ is a vector of ones. $\langle x, y\rangle$ is the standard inner product for vectors x and y, while $|x|$ and $|x|_\infty$ are respectively the 2-norm and infinity norm of x. For a matrix M, M' denotes its transpose and $|M|$ its induced 2-norm. $\mathrm{diag}\{a_1, \ldots, a_n\}$ denotes a diagonal matrix with diagonal entries given by $a_1, \ldots, a_n$. $\mathrm{col}\{x_1, \ldots, x_n\} = [x_1', \ldots, x_n']'$. $\sum_i$ denotes a summation for all possible index i, which similarly applies to $\max_i$ and $\min_i$. $f(\varepsilon) = O(g(\varepsilon))$ and $f(\varepsilon) = o(g(\varepsilon))$ respectively mean that $\limsup_{\varepsilon\to 0} \frac{|f(\varepsilon)|}{|g(\varepsilon)|} \leq C$ with some constant $C > 0$ and $\lim_{\varepsilon\to 0} \frac{|f(\varepsilon)|}{|g(\varepsilon)|} = 0$.

2. PRELIMINARIES

2.1 Graph Theory

The two-way communication between different agents within a multi-agent system can be modeled as an undirected graph $\mathcal{G}$, which includes a node set $\mathcal{N} = \{1, \ldots, N\}$ and an edge set of unordered pairs $\mathcal{E} = \{(i, j) :\ i, j \in \mathcal{N}\}$ excluding self-loop (i, i). $(i, j) \in \mathcal{E}$ indicates a mutual transmission channel between node j and i, and $\mathcal{N}_i = \{j \mid j \in \mathcal{N},\ (j, i) \in \mathcal{E}\}$ denotes the neighbor set of node i. $\mathcal{G}$ is said to be connected if there always exists a path between two different nodes i and j, which is given as an ordered edge sequence $(i, n_1), \ldots, (n_j, j)$. A nonnegative matrix $A = (a_{ij}) \in \mathbb{R}^{N\times N}_{\geq 0}$ can be assigned as the weights on the edges, with $a_{ij} = a_{ji} > 0$ iff $(j, i) \in \mathcal{E}$. The triplet $\{\mathcal{N}, \mathcal{E}, A\}$ completely defines a weighted graph $\mathcal{G}$. The corresponding Laplacian matrix is $L = D_{\mathcal{G}} - A$, where $D_{\mathcal{G}} = \mathrm{diag}\{D_1, \ldots, D_N\}$ with $D_i = \sum_{j\in\mathcal{N}_i} a_{ij}$. $\mathcal{G}$ is connected iff the N eigenvalues of L can be rearranged as $0 = \lambda_1 < \lambda_2 \leq \cdots \leq \lambda_N$.

2.2 Convex Functions

A function $f : \mathbb{R}^m \to \mathbb{R}$ is convex if the inequality

$$f(ax + (1 - a)y) \leq af(x) + (1 - a)f(y)$$

holds for any x, y and $0 \leq a \leq 1$. If f is further differentiable with gradient ∇f, then we have $f(y) - f(x) \geq \langle\nabla f(x), y - x\rangle$. f is called ω-strongly convex if

$$\langle\nabla f(x) - \nabla f(y), x - y\rangle \geq \omega|x - y|^2,\ \forall x, y, \tag{1}$$

where $\omega > 0$ is a constant. Note that (1) is equivalent to

$$f(y) \geq f(x) + \langle\nabla f(x), y - x\rangle + \frac{1}{2}\omega|x - y|^2,\ \forall x, y. \tag{1'}$$

3. PROBLEM FORMULATION

Consider N agents with the following EL dynamics:

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) = \tau_i,\ i = 1, \ldots, N, \tag{2}$$

where $q_i, \dot{q}_i \in \mathbb{R}^m$ denote the generalized position and velocity vectors respectively; $M_i \in \mathbb{R}^{m\times m}$ is the positive definite inertia matrix dependent on the position; $C_i(q_i, \dot{q}_i)\dot{q}_i$ is the Coriolis and centripetal forces; G_i is the gravity, and τ_i is the control force. The following assumption is commonly adopted, e.g. Spong and Vidyasagar (2008):

Assumption 1. $|C_i(q_i, \dot{q}_i)| \leq C|\dot{q}_i|$ with $C \geq 0$.

Moreover, each agent i is assigned with an individual cost f_i, and we aim to design a distributed control law for all agents to cooperatively minimize the aggregate cost:

$$\min\nolimits_{q\in\mathbb{R}^m} f(q) = \sum\nolimits_i f_i(q), \tag{3}$$

or equivalently

$$\min\nolimits_{q_i\in\mathbb{R}^m} \sum\nolimits_i f_i(q_i),\ \text{s.t. } q_1 = \cdots = q_N. \tag{3'}$$

Therefore, if we denote the optimum of (3) as q^*, then we shall prove that

$$\lim\nolimits_{t\to\infty}(q_i - q^*) = 0,\ i = 1, \ldots, N. \tag{4}$$

Furthermore, for practical implementation the velocity and control force are required to be bounded. Specifically, we consider the case when the gravity can be ignored or effectively canceled, say respectively in the case of ground robots or UAVs. Hence we define the bounded constraints on $\dot{q}_i$ and $\tilde{\tau}_i = \tau_i - G_i$ as

$$|\dot{q}_i|_\infty \leq \overline{V},\ |\tilde{\tau}_i|_\infty \leq \overline{U}. \tag{5}$$

Below we make two assumptions respectively about the communication topology and the individual cost functions.

Assumption 2. The communication graph $\mathcal{G}$ is connected.

Assumption 3. For each i, f_i is differentiable and ω-strongly convex, and ∇f_i is θ-Lipschitz, namely

$$|\nabla f_i(x) - \nabla f_i(y)| \leq \theta|x - y|,\ \forall x, y.$$

4. MAIN RESULTS

In this section we are to study the optimal consensus (4) of multiple EL systems under constraints (5). Noticing that any EL system with exact knowledge of nonlinearities can be transformed into a double-integrator system, we first explore the optimal consensus under kinematic constraints for double-integrator multi-agent systems. Then we proceed to the EL case by employing the inverse dynamics control. See the following two subsections for details.