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Transient Stability Analysis of Microgrids with Network-Preserving Structure

Lijun Zhu * **David J. Hill** *,**

* *Department of Electrical and Electronic Engineering, The University of Hong Kong, Hong Kong. (e-mail: ljzhu,dhill@eee.hku.hk).*
** *Department of Electrical and Information Engineering, The University of Sydney, Australia.*

Abstract: The angle stability of a power system typically refers to its ability to remain in synchronism when severe failures or faults occur. This stability definition characterizes the capability of the post-fault power system returning to a system equilibrium after disrupting events. In fact, power system experiences a variety of disturbances and fluctuations in load and generation around the nominal operation conditions even in the absence of serve system faults. Taking these issues into account, this paper extends the definition of stability and considers the transient stability of microgrids equipped with inverter-based energy sources. The model of microgrids is presented in a network-preserving fashion without conducting network reduction and hence the irregularity of the physical power network topology is retained. The objective of the paper is to investigate the impact of the underlining network topology and system parameters on the stability of the power system. We propose a new analysis method that enables this exploration and derives a simple algebraic condition that relates the stability, system dynamics and network topology.

Keywords: Microgrid, Transient Stability, Network-preserving Model, Network Topology.

1. INTRODUCTION

Microgrids are low-voltage electrical distribution networks consisting of distributed energy sources and loads (see, e.g., Simpson-Porco et al. (2012, 2013); Lasseter (2002)). The energy sources in the microgrids, usually renewable energy such as solar, wind power, and geothermal system, are fed via inverters with droop controllers implementing frequency and voltage regulation. Since the inverter-based energy sources have zero inertia, the stability behavior of islanded microgrids is different from traditional power grids equipped with conventional synchronous machine based power generators. Therefore, the stability of islanded microgrids needs to be considered independently.

Power system stability has been long recognized as an important issue for stable and secure system operation in order to deliver electric power reliably from generators to loads. The angle stability of a power system typically refers to its ability to remain in synchronism after severe failures or faults such as short-circuits in transmission lines. The stability analysis concerns the capability of the post-fault power system returning to the system equilibrium after disrupting events. Over the last decade, modern power system has seen increasingly large amounts of renewable energy being integrated into grids. The power supplied by renewable energy has stochastic and intermittent nature, when fed into power grids, causing fluctuations in the total power generation. The real-time power demand, which is usually predicted ahead, is in fact also instantaneously fluctuating around predicted values. Unlike disrupting events that occur to power systems occasionally, the fluctuations and so-caused mismatch in load and generation are taking place along the course of system operation. These issues bring about a variety of challenges to the stability of power systems and have yet to be taken into explicit account. The system's ability and tolerance to withstand the disturbances and fluctuations represent the robustness of power systems while operating at a nominal condition.

Many early works such as Athay et al. (1979); Lüders (1971) used a circuit-theoretic method called Kron reduction to derive the dynamical model for power systems. The Kron reduction reduces the original power network into a network of generators connected in an all-to-all fashion and brings about two drawbacks: inclusion of transfer conductance and the loss of the network topological information. The former makes developing general Lyapunov functions for stability analysis unsuccessful, while the latter prevents the study of the relation between the original network topology and transient stability. To tackle these problems for conventional power systems, Bergen and Hill (1981) proposed the network-preserving model of the power systems and studied the transient stability using Lyapunov functions in Lur'e-Postnikov form (Bergen and Hill (1981); Hill and Bergen (1982); Hill and Chong (1989). Following Bergen and Hill (1981)), the network-preserving model of microgrids equipped with inverter-based energy sources was first proposed in Ainsworth and Grijalva (2013) where the frequency synchronization condition was studied. The small disturbance stability of microgrids has been studied

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by Song et al. (2015); Ainsworth and Grijalva (2013). The transient stability of the microgrid remains to be completely solved.

In this paper, we consider transient stability for the microgrid modeled with a network-preserving structure. The main objective and contribution of this paper are summarized as follows. First, a new definition for the stability of power systems will be introduced and the stability criterion is characterized by the angle differences across all transmission lines. Also, the definition accommodates stability analysis under the time-varying instantaneous disturbances. Second, we introduce a Lyapunov function which is inspired by the topological Lyapunov function in Bergen and Hill (1981) for the stability analysis. Since this Lyapunov function only considers the potential energy induced across the transmission lines, it enables studying the stability in the sense of the new definition. Finally, we derive a concise algebraic condition for the synchronization that is related to the network topology and system parameters.

The rest of the paper is structured as follows. Section 2 presents the structure-preserving model of microgrids, introduces the new stability definition and describes the problem to be studied. In Section 3, we first rephrase the problem with respect to the stable equilibrium point of power systems, propose the Lyapunov function to study the transient stability, and obtain the synchronization condition. Section 4 verifies the theoretical results on the IEEE 9-bus test system using numerical simulation. The paper is concluded in Section 5.

2. SYSTEM MODEL AND PROBLEM FORMULATION

The network-preserving model of microgrids equipped with inverter-based energy sources was first proposed in Ainsworth and Grijalva (2013) where the transmission lines are assumed to be lossless. The network-preserving structure was inspired by the early work Bergen and Hill (1981) which was focused on classical power grids. In this paper, we consider that the microgrid consists of total n buses with n_r energy sources and n_l load buses. Denote by a_{ij} the maximum electrical energy that can be transmitted between bus i and j, and p_i the energy produced at buses of energy sources or energy consumption at load buses. Buses for energy sources and loads can be unified as the first-order dynamics

$$d_i\dot{\theta}_i = p_i - \sum_{j=1}^{n} a_{ij}\sin(\theta_i - \theta_j),\ i = 1, \cdots, n. \quad (1)$$

where d_i represents different physical quantities for different types of buses (see Simpson-Porco et al. (2012) and Ainsworth and Grijalva (2013) for a more detailed derivation)

The dynamical system in (1) coincides with the celebrated Kuramoto oscillators but with irregular network topology (not all-to-all connection). Dörfler and Bullo (2012) linked Kuramoto oscillators with the stability analysis of power systems where the synchronization-like concepts such as phase cohesiveness and frequency synchronization were introduced. In this paper, we adapt this definition to describe the stability of microgrids as follows.

Definition 2.1. (Synchronization: Phase and Frequency Boundedness). Let $\theta = [\theta_1, \cdots, \theta_n]^{\mathrm{T}} \in \mathbb{R}^n$ be the angle vector. A solution $\theta(t) : \mathbb{R}^+ \to \mathbb{R}^n$ is then said to be phase cohesive if there exists a $\gamma \in [0, \pi)$ such that $\max_{(i,j)\in\mathcal{E}} |\theta_i - \theta_j| \leq \gamma$. A solution $\dot{\theta}(t) : \mathbb{R}^+ \to \mathbb{R}^n$ is then said to be frequency boundedness if there exists a $\varpi_o \in \mathbb{R}$ such that $\|\dot{\theta}(t)\| < \varpi_o$. ∎

In Dörfler and Bullo (2012), phase cohesiveness condition $\max_{(i,j)\in\mathcal{E}} |\theta_i - \theta_j| \leq \gamma$ in Definition 2.1 is replaced by $\max_{i,j=1,\cdots,n,i\neq j} |\theta_i - \theta_j| \leq \gamma$ which takes angle differences of any two buses into account including those not physically connected by transmission lines. However, Definition 2.1 only considers the angle differences across physical transmission lines.

The objective of this paper is to investigate synchronization in the sense of Definition 2.1 for microgrids described by the dynamical system (2). Inspired by Dörfler and Bullo (2012), we will explore the influence of the network structure on the synchronization for microgrids and established a pure algebraic synchronization condition. Moreover, we extend our analysis to the case of time-varying power disturbance. The algebraic graph theory is employed here. An undirected $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consists of a set of vertices $\mathcal{V} = \{1, \cdots, n\}$ and a set of undirected edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. An undirected edge of $\mathcal{E}$ from node i to node j is denoted by (i, j), meaning that nodes $\mathcal{V}_i$ and $\mathcal{V}_j$ are interconnected with each other. The edge weight is denoted by a_{ij} where $a_{ii} = 0$ and $a_{ij} = a_{ji} > 0$ for $(j, i) \in \mathcal{E}$. The Laplacian of the graph $\mathcal{G}$ is denoted by $L = [l_{ij}] \in \mathbb{R}^{n\times n}$, where $l_{ii} = \sum_{j=1}^{n} a_{ij}$ and $l_{ij} = -a_{ij}$ if $i \neq j$. Denote by $\mathcal{E}_k$ the kth edge of $\mathcal{E}$ where $k \in \{1, \cdots, |\mathcal{E}|\}$, $|\mathcal{E}|$ the number of edges, and $B \in \mathbb{R}^{n\times|\mathcal{E}|}$ the incidence matrix whose component is $B_{ik} = 1$ if node i is the sink node of edge $\mathcal{E}_k$, $B_{ik} = -1$ if it is the source node and $B_{ik} = 0$ otherwise. As a result, one can have $L = BA_vB^{\mathrm{T}}$ where $A_v = \mathrm{diag}(\{a_{ij}\}_{(i,j)\in\mathcal{E}})$ is the diagonal matrix with diagonal elements being edge weights. For an n-bus microgrid, the vertices $\mathcal{V} = \{1, \cdots, n\}$ represent the buses and the edges $(i, j) \in \mathcal{E}$ represent the transmission lines.

Notations. For a scalar $x \in \mathbb{R}$, $\mathrm{sinc}(x) = \sin(x)/x$. For a vector $x = [x_1, \cdots, x_n]^{\mathrm{T}} \in \mathbb{R}^n$, $\|x\|$ and $\|x\|_\infty$ are the 2-norm and the ∞-norm of vector x and $\sin(x) := [\sin(x_1), \cdots, \sin(x_n)]^{\mathrm{T}}$. The vector e_n is a column vector of dimension n with all elements being 1.

3. STABILITY ANALYSIS

The dynamical system of microgrids (1) can be put in a concise form. Let the coefficient matrix be $D = \mathrm{diag}(d_1, \cdots, d_n)$ and let us call the vector of the load demands and power generations $p_f = [p_1, \cdots, p_n]^{\mathrm{T}} \in \mathbb{R}^n$ the power profile. Then, the dynamical system (1) in the vector form is given as follows

$$\dot{\theta} = -D^{-1}\left(BA_v\sin(B^{\mathrm{T}}\theta) - p_f\right) \quad (2)$$

where B is the incidence matrix of the power network $\mathcal{G}$.

3.1 Problem Conversion

Let us first consider the equilibrium point of the dynamical system (2). Power systems such as microgrids are normally