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Piecewise Bézier Curve Fitting by Multiobjective Simulated Annealing

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Abstract: The determination of an approximation curve from a given sequence of points is an important task in CAD. This work proposes an algorithm to determine a piecewise Bézier curve that approximates a sequence of points. It is used a multiobjective simulated annealing aiming at minimizing the discrepancy between the given sequence of points and the curve, the curve length and the absolute difference of the curve length and length of the given sequence of points. The discrepancy between the given sequence of points and the curve is determined by the sum of the distance between each point from the sequence and the approximation curve, and the distance from a point to the curve is determined by an enhanced method in which the curve is discretized.

Keywords: Bézier Curve, Piecewise Curve, Curve Fitting, Multiobjective Simulated Annealing, CAD.

1. INTRODUCTION

One task very important in engineering is to determine a curve from a given sequence of points, specially in reverse engineering, where it can be used to reconstruct objects. There are methods in CAD that uses curves to create surfaces (Piegl and Tiller, 1996), and by determining these curves the object can be reconstructed. Another application is the path planning by determining points (places) where the vehicle had to pass through.

The task to determine a curve from a sequence of point can be done by two different approaches, one by interpolation (Maekawa et al., 2007; Gofuku et al., 2009; Jakóbczak, 2015) and another by approximation (Pandunata and Shamsuddin, 2010; Hasegawa et al., 2013; Adi et al., 2010). The first determines a curve or function that pass by all the points, while the second determines a curve that is closer to a sequence of points. This work presents a new method of curve approximation that uses piecewise Bézier curves and a multiobjective simulated annealing to approximate a sequence of points. This work is structured as follow. In section 2 is done a brief review of Bézier curve, the curve fitting problem and piecewise cubic Bézier curve. In section 3 is presented all the cost functions to be minimized as well as the problem parametrization. In section 4 the results and finally a conclusion in section 5.

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2. BÉZIER CURVE

A Bézier curve is given by

$$\mathbf{P}(u) = \sum_{i=0}^{n} \mathbf{p}_i B_{i,n}(u), \qquad u \in [0,1] \tag{1}$$

where $\mathbf{p}_i$ are the control points, $n+1$ the total numbers of control points and $B_{i,n}(u)$ is the Bernstein polynomial basis function that is defined as

$$B_{i,n}(u) = \binom{n}{i} u^i (1-u)^{n-i}, \qquad i = 0, ..., n. \tag{2}$$

where $\binom{n}{i}$ is the binomial function, given by

$$\binom{n}{i} = \frac{n!}{i!(n-i)!} \qquad \binom{0}{0} \equiv 1. \tag{3}$$

2.1 Piecewise Bézier Curve Segments

The characteristic of each control point of a Bézier curve modify all the curve can be avoided by using piecewise cubic Bézier curve segments. This sequence of curve segment is done with a weak C^1 continuity with the surrounding curves, where the derivatives have the same direction, but not necessary the same intensity. To create these sequence of curves, the last control point of a curve is the first control point of the following curve, and to determinate the second control point of a curve segment it is used the equation

$$\mathbf{p}_{3i+4} = \mathbf{p}_{3i+3} - (\mathbf{p}_{3i+2} - \mathbf{p}_{3i+3}) * \beta_{3i+4}, \tag{4}$$

where $i = 0, \ldots, j-1$ and $j-1$ is the number of segments and β_i is a continuity factor. An example is shown in Fig. 1,

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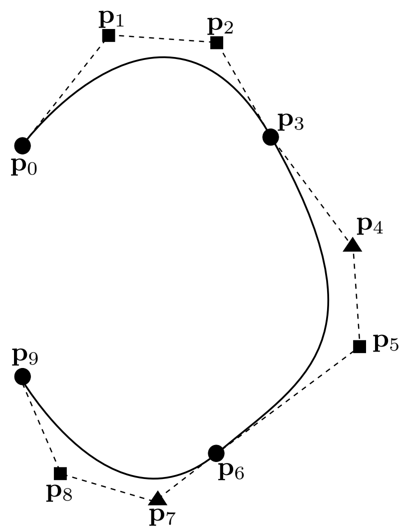

Fig. 1. Sequence of three cubic Bézier curve. Points $\mathbf{p}_3$ and $\mathbf{p}_6$ are transition points between Bézier curves.

that is a composition of three segments of cubic Bézier curve.

2.2 Bézier curve fitting

The objective of curve fitting problem is to define a curve that approximates a given sequence of points by determining its control points $(\mathbf{p_0}, ..., \mathbf{p_n})$. This determination is done by minimizing the discrepancy between the curve and the given sequence of points, which can be calculated as the distance between the curve and each point of the given sequence of points. The function that describes the function to be minimized is

$$f(\mathbf{p}_0, ..., \mathbf{p}_n) = \sum_{k=1}^{m-1} |\mathbf{d}_k - \mathbf{P}(u_k)|^2 \tag{5}$$

where $0 < k < m$, $n+1$ is the number of control points and d is the sequence of points with $m+1$ elements. $|\mathbf{d}_k - \mathbf{P}(u_k)|$ is the distance between one point $\mathbf{d}_k$ and the Bézier curve. u_k is the parameter value that defines which point of the curve is the closest to the point $\mathbf{d}_k$, which also can be interpreted as the projection of the point $\mathbf{d}_k$ over the curve. Points $\mathbf{d}_0$ and $\mathbf{d}_m$ are not added in the sum, once the Bézier curve always interpolates the first and last control points.

There are several methods to determine the parameter u_k. Hasegawa et al. (2013) and Pandunata and Shamsuddin (2010) determined this parameter in an approximated approach by using the chord length parametrization. This parametrization approximates the u_k parameter as a fraction between the sum of the chords until the respective point $\mathbf{d}_k$ and the sum of all the chords. Maekawa et al. (2007) used the Newton-Raphson method to determine the parameter u_k which makes the segment that connect the points $\mathbf{P}(u_k)$ and $\mathbf{d}_k$ orthogonal to the tangent at $\mathbf{P}(u_k)$ point. Tavares et al. (2011) used a method in which the curve is discretized with a sequence of points and all the points are measured to the given point $\mathbf{d}_k$ and it is determined the point of the discretized curve that is closer to $\mathbf{d}_k$.

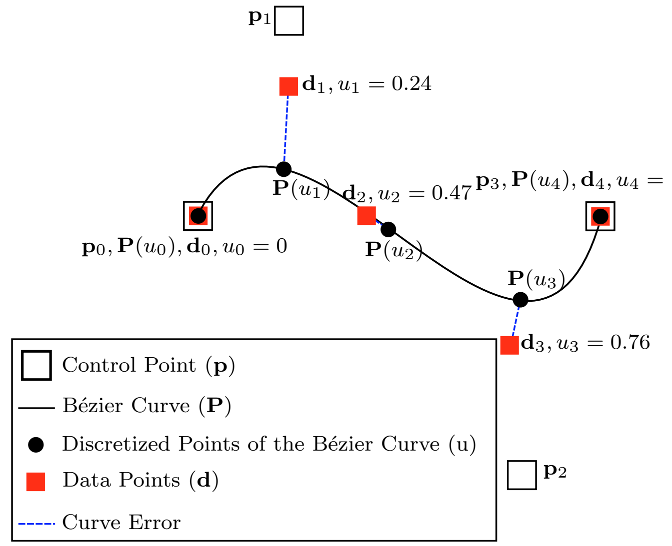

Fig. 2. Distance determined by an approximated method.

The methods proposed by Hasegawa et al. (2013) and Pandunata and Shamsuddin (2010) could wrongly determine the points as shown in Fig. 2. Maekawa et al. (2007) has some advantages, although its processing cost is higher. On the other hand, the method proposed by Tavares et al. (2011) is the one with more advantages due to the balance between precision and processing time. In this method, the higher the discretization the better the quality and the processing time increases.

3. COST FUNCTIONS

Equation (5) is the first cost function adopted. This expression evaluate just the discrepancy between curve and given points. Other functions could be evaluated. In this work it will be also evaluated the curve length and the absolute difference between the length of the curve and the given sequence of points. Once (5) defines a family of curves, the other two functions will evaluate the difference among the curves of the family.

The method to determine the discrepancy between point and curve is an enhancement of the method proposed by Tavares et al. (2011). In the determination of the point of a discretized curve that is closer to a given point $\mathbf{d}_k$, there are considered two triangles as shown is Fig. 3. Squares points are points of the discretized curve and circle point is a point of the given sequence of points.

Firstly, a search is done to determine which point of the discretized curve is closer to the point of the sequence $\mathbf{d}_k$. This point is named $\mathbf{P}(u_k)$. As shown in Fig. 3, two triangles are defined. The first is defined by the points $\mathbf{d}_k$, $\mathbf{P}(u_k)$ and $\mathbf{P}(u_{k-1})$; the second is defined by $\mathbf{d}_k$, $\mathbf{P}(u_k)$ and $\mathbf{P}(u_{k+1})$.

Then it is evaluated which triangle has the smallest area. Next, it is made a verification whether the triangle with smallest area is acute or obtuse, in specific the angles between the segments $\overline{\mathbf{d}_k\mathbf{P}(u_k)}$ and $\overline{\mathbf{P}(u_k)\mathbf{P}(u_{k-1})}$ for the first triangle, and $\overline{\mathbf{d}_k\mathbf{P}(u_k)}$ and $\overline{\mathbf{P}(u_k)\mathbf{P}(u_{k+1})}$ for the second. If the triangle with the smallest area is acute, it is calculated the height of the triangle related to the segment that is defined by two points of the discretized curve, and