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# Study of the grid size impact on a raster based strip packing problem solution

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**Abstract:** Cutting and packing (C&P) is an important area of operational research and its problems arise in various industries such as: textile, wood, glass and shipbuilding. The main objective is to maximize the efficiency of a layout by rearranging and/or reassigning items inside containers in order to reduce costs and environmental impact. In this work, a raster solution to the bidimensional irregular strip packing problem, which consists of placing irregular shapes items inside a single rectangular container with variable length, is studied. In raster methods, the selection of the grid size is very important to the outcome of the algorithm. It influences the size of the search space, the overlap algorithm efficiency, as well as the memory requirements of the packing algorithm. An analysis of the impact of the choice of grid size is performed using 15 benchmark cases from the literature and, through careful observation of such test results, a simple rule to define the grid size is suggested.



*Keywords:* Irregular strip packing, overlap minimization, raster method

## 1. INTRODUCTION

The cutting and packing research area consists of essentially waste minimization problems concerning layout generation. It usually involves the arrangement and/or assortment of items inside one or more containers. An improved solution usually incurs in significant economical and environmental benefits. These problems arise in industries such as textile, wood, glass and shipbuilding.

Due to its importance in textile industry, two dimensional irregular strip packing is a well researched sub-problem in the C&P area. In such case, a single rectangular container with fixed width must accommodate a set of irregular shaped items, which may admit up to four orientations. The objective is to find a layout with minimum length, maximizing its compaction (see Fig. 1). The solution must be a feasible layout, i.e. there must be no overlap between items and they must all lie completely inside the container.

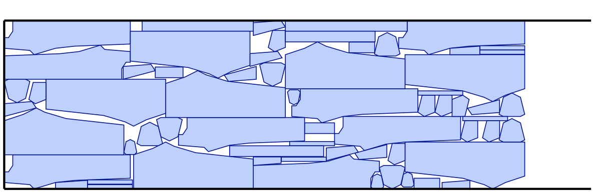

Fig. 1. Solution for a irregular strip packing problem in the textile industry.

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Packing problems usually deal with regular shapes, such as rectangles, circles or cubes. However, in the textile industry, irregular shaped patterns are cut from fabrics and, thus, a more complex geometric tool is needed to process the overlap. The difficulty of handling the irregular shapes is one of the main challenges of this group of problems.

There are basically two types of approaches in the literature for the geometry representation in irregular strip packing problems: vector and raster. Vector representation consists of describing the geometry as sequence of vertexes, whereas a raster representation employs a pixel based grid. Raster representations simplify collision detection in packing problems. However, it is intrinsically less precise when compared to geometric overlap detection tools. Such property restricts a more widespread use of raster methods in the literature.

In this work, raster techniques are employed with geometric represented items to solve the irregular strip packing problem using the dotted board model. In such approach, item placements are limited by a grid inside the container. Even with this strong restriction, tests show that, depending on the grid size, the results are competitive with other approaches in the literature. Given its importance to the adopted approach, an analysis of the influence of the grid size is performed in order to determine a better method to select the rasterization parameter.

This paper is structured as follows. Section 2 gives a brief summary of more recent approaches to solving the irregular strip packing problem in the literature. In section 3, the overlap minimization approach is discussed and,


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in conjunction with the description in section 4, convey the basic concepts of the adopted methodology. Section 5 contains a more detailed discussion on the rasterization problem, which is the main novel characteristic of the discussed solution. The description of the tests and an analysis of the influence of the rasterization is given in section 6 and conclusions are drawn in section 7.

## 2. LITERATURE REVIEW

The irregular strip packing problem is generally viewed as an optimization problem. The objective is to find the most compact layout given a set of items and a single container with a fixed width and variable length. The problem has two main restrictions: 1) no two items can overlap and 2) all items must be placed completely inside the container.

A variant formulation can be obtained by relaxing the item overlap restriction and setting a fixed value for the length of the container. Thus, during the solution searching process, items are allowed to move freely inside the container and the objective is to eliminate the overlap. Once resolved, the problem is solved with a smaller container. Hence, the layout is iteratively compacted. This type of solution, commonly alluded to as overlap minimization approach, is adopted by the more successful approaches.

Egeblad et al. (2007) proposed an algorithm which evaluated overlap when items were translated horizontally along a straight line, and combined it with a guided neighborhood search. This approach was further improved by Umetani et al. (2009), which allowed alternating horizontal and vertical item movements. It used a modified guided local search to obtain the compacted layout. Such metaheuristic was also adopted by Imamichi et al. (2009) and Leung et al. (2012). In both approaches, nonlinear programming was employed to minimize the layout overlap, translating all items simultaneously. Elkeran (2013) obtained the best solution up to date by a combination of the guided local search and a cuckoo search to find the best minimum overlap positions for items.

All cited solutions in the literature use a vector representation of the items. There are few approaches which employ a raster, or pixel, item representation, mostly due to its limited precision. Segenreich and Braga (1986) uses a matrix to represent the items. Each cell contains a value to indicate whether the point is inside, outside or along the contour of the polygon. Babu and Babu (2001) fills the item matrix with distance values. These corresponds to the minimal horizontal distance to the items contour. These values can be used efficiently to separate items. Among the advantages of the raster methods is its generic approach to geometry, as items may have arbitrary shape, and very fast overlap calculations.

Sato et al. (2014) proposed the use of raster techniques to a overlap minimization algorithm, which uses vector represented items. It aimed to achieve the performance of the raster approaches with the compaction potential of overlap minimization methods. The algorithm achieved very good results. Nevertheless, it was possible to observe that the quality of results heavily depended upon the choice of grid size, which is not straightforward to determine. Therefore, in this work, a exhaustive grid size study is conducted for a more deep analysis of such method's capabilities.

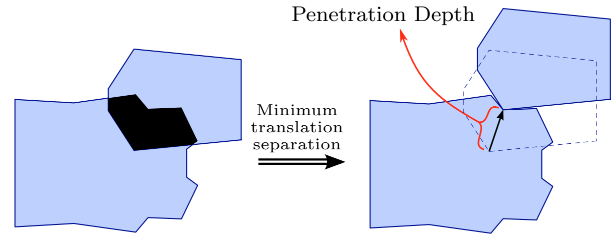


Fig. 2. Example of penetration depth determination.

## 3. OVERLAP MINIMIZATION

In the approach adopted in this work, overlap is allowed during the execution of the algorithm. In the final layout, however, all geometric restriction must be satisfied. In order to relax the collision condition, a value must be assigned to evaluate the overlap. In this work, the penetration depth is adopted as the overlap function and the nofit polygon geometric tool is employed to determine its value. In order to minimize the overall overlap value of a layout, a separation approach is performed.

### *3.1 Penetration Depth and Nofit Polygon*

The penetration depth is the measure of the minimum distance one of the items in a pair must be translated in order to fully eliminate the overlap (see Fig. 2). If the two items are separated, its value is zero. Due to its properties, the penetration depth is adopted in many overlap minimization solutions in literature (Imamichi et al., 2009; Elkeran, 2013; Sato et al., 2014) as the overlap function. Consider a pair of items $P_i$ and $P_j$, with established positions in space and fixed orientations. The penetration depth $\delta(P_i, P_j)$ for the item pair $P_i$ and $P_j$ can be described as

$$\delta(P_i, P_j) = \min\{\|v\| \mid i(P_j \oplus v) \cap i(P_i) = \emptyset\} \quad (1)$$

where $\|\cdot\|$ denotes the Euclidean norm.

The determination of the penetration depth can be accomplished using the nofit polygon concept. The nofit polygon is the most popular geometric tool for irregular packing problems. Proposed by Art (1966), it maps, for a pair of items, all possible overlap configurations. It can be represented by a region in space by setting a reference point to one of the items, as seen in the example in Fig. 3.

The nofit polygon reduces the complexity of determining whether two items are overlapped by performing most of the calculation prior to the collision detection. The final evaluation consists of determining if the reference point is inside the nofit polygon. If positive, the items are overlapping; otherwise, the configuration is valid.

The penetration depth is equivalent to the minimum distance from the reference point to the contour of the corresponding nofit polygon.

### *3.2 Layout Overlap evaluation*

The objective of an algorithm that solves the irregular strip packing problem is to obtain the most compact