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Global Regulation of Time-Delay Cascade Systems via Partial State Feedback ⋆

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Abstract: The problem of global stabilization is studied by partial state feedback for a class of cascade systems with time-delay. Under suitable ISS conditions on inverse dynamics, a delay-free, dynamic partial state feedback compensator is designed to achieve global state regulation. This is made possible by developing a dynamic gain based design method, together with the ideas of changing supply rates and backstepping. By constructing appropriate Lyapunov-Krasovskii functionals, we prove that all the states of the time-delay cascade system can be regulated to the origin while maintaining boundedness of the closed-loop system.

1. INTRODUCTION

In this paper, we consider the time-delay cascade system

$$\begin{aligned}\dot{z}_i &= f_{0i}(z_1, \cdots, z_i, z_1(t-d), \cdots, z_i(t-d),\\ &\quad x_1, \cdots, x_i, x_1(t-d), \cdots, x_i(t-d)),\\ \dot{x}_i &= x_{i+1} + g_i(z_1, \cdots, z_i, z_1(t-d), \cdots, z_i(t-d),\\ &\quad x_1, \cdots, x_i, x_1(t-d), \cdots, x_i(t-d)),\\ z_i(s) &= \zeta_i(s), \quad x(s) = \mu(s), \qquad s \in [-d, 0],\\ &\quad i = 1, \cdots, r, \end{aligned} \tag{1}$$

where $z_i \in \mathbb{R}^{\mathrm{n_i}} (\mathrm{n_i} = 0, 1, 2, \cdots)$ and $x = [x_1, \cdots, x_r]^T \in \mathbb{R}^{\mathrm{r}}$ are the system states, $u := x_{r+1} \in \mathbb{R}$ is the control input, and the constant $d \geq 0$ is an unknown time-delay of the system. For $i = 1, \cdots, r$, $f_{0i} : \mathbb{R}^{2\mathrm{n_1}+\cdots+2\mathrm{n_i}+2\mathrm{i}} \to \mathbb{R}^{\mathrm{n_i}}$ and $g_i : \mathbb{R}^{2\mathrm{n_1}+\cdots+2\mathrm{n_i}+2\mathrm{i}} \to \mathbb{R}$ are C^1 mappings with $f_{0i}(0) = 0$ and $g_i(0) = 0$, and $\zeta_i(s) \in \mathbb{R}^{\mathrm{n_i}}$ and $\mu(s) \in \mathbb{R}^{\mathrm{r}}$ are continuous functions defined on $[-d, 0]$. Notably, dim z = dim $[z_1, \cdots, z_r]^T = n_1 + n_2 + \cdots + n_r$ with $n_i \geq 0$, which can be either less or bigger than, or equal to dim $x = r$. In this work it is assumed that only the partial state $x = [x_1, \cdots, x_r]^T$ of the cascade system (1) is measurable and available for feedback design.

In the absence of time-delay, control of the cascade nonlinear system (1) has been studied by partial state feedback in Lin and Pongvuthithum [2002b], Lin and Radom [2003], Lin and Gong [2003], Chen and Huang [2004], Chen [2009] and the references therein. In the case when only z_1 appears in the cascade system (1), the problem of global stabilization was considered in Isidori [1999], Lin and Pongvuthithum [2002b], Lin and Gong [2003] by using small-gain theorem, and the idea of changing supply rate combined with the backstepping design. In Chen and Huang [2004], Chen [2009], global stabilization of the more general cascade system such as (1) without time-delay was shown to be possible by using a Lyapunov direct method. It was pointed out in Chen and Huang [2004] that the states $(z_1, \cdots, z_r)$, which model the dynamic uncertainty, represent the internal model of the cascade system when studying the robust output regulation of lower-triangular systems. The work Chen [2009] provided an explicit construction of a Lyapunov function in superposition form for the cascade nonlinear system.

Systems with time-delay are frequently encountered in practice and often causes instability. For time-delay systems, many results have been obtained and reported in the literature Richard [2003], Krstic [2010], Gu, Kharitonov, and Chen [2003], Pepe [2014]. However, little progress has been made for the control of the cascade system (1) with time-delay. Even in the case when the nonlinear system (1) contains no dynamic uncertainty z_i, the stabilization problem is still a difficult one as shown in Jankovic [2001], Mazenc, Mondie, and Niculescu [2003], Karafyllis and Jiang [2010], Zhang and Lin [2014].

Following the work Chen [2009], we study in this paper the problem of how to control the time-delay nonlinear cascade system (1) by delay-independent, partial state feedback. Another motivation of our investigation is due to the work Chen and Huang [2004] on global output regulation of lower-triangular systems with zero dynamics. It has indicated that in the absence of time-delay, there is a closed connection between the solvability of the global output regulation problem and the global stabilizability of cascade nonlinear systems by partial state feedback. Therefore, the study of the cascade nonlinear system (1) with time-delay is likely to provide a new insight in understanding the global output regulation of time-delay nonlinear systems and may pave a way to solve the global output regulation problem.

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The main contribution of this paper is to show that global state regulation of the time-delay cascade system (1) is possible by *delay-free, dynamic partial state* feedback if the inverse dynamics satisfy appropriate ISS conditions. Specifically, one can design a delay-independent, partial state feedback controller with dynamic gains that are updated by Riccati-like equations. The constructed partial state dynamic compensator can globally regulate all the states of the time-delay cascade system (1) to the origin while maintaining the boundedness of the resulted closed-loop system. The novelty lies in the development of a dynamic partial state feedback control strategy based on the backstepping design and changing supply rates, capable of counteracting the time-delay nonlinearities of the cascade system (1). Another new ingredient of this work is the construction of appropriate Lyapunov-Krasovskii functionals that relies on dynamic gains, which play a crucial role in proving the global stability as well as the state regulation of the cascade system with time-delay.

Notations: In this paper, let v_d denote the time-delay term $v(t-d)$, for example, $z_{id} = z_i(t-d)$ and $x_{id} = x_i(t-d)$. Define $\bar{v}_i = [v_1, \cdots, v_i]^T \in \mathbb{R}^{\mathrm{i}}$ for $i = 1, \cdots, r$. Hence, $\bar{x}_i = [x_1, \cdots, x_i]^T$, $\bar{x}_{id} = [x_{1d}, \cdots, x_{id}]^T$ and $\bar{l}_i = [l_1, \cdots, l_i]^T$.

2. USEFUL LEMMAS

The following two lemmas are very useful when designing a dynamic partial state feedback compensator for the time-delay cascade system (1).

Lemma 2.1 Lin and Qian [2002a] Let $x \in \mathbb{R}^{\mathrm{n}}$, $y \in \mathbb{R}^{\mathrm{m}}$ and $f : \mathbb{R}^{\mathrm{n}} \times \mathbb{R}^{\mathrm{m}} \to \mathbb{R}$ be continuous. Then, there are smooth scalar functions $a(x) \geq 0$, $b(y) \geq 0$, $c(x) \geq 1$ and $d(y) \geq 1$, such that

$$|f(x,y)| \leq a(x) + b(y), \quad |f(x,y)| \leq c(x)d(y). \tag{2}$$

Lemma 2.2 Zhang and Lin [2014] If $f(x,y)$ is a real-valued continuous function, there exist smooth scalar functions $g(x) \geq 0$ and $h(y) \geq 0$ satisfying

$$f(x,y)(\|x\| + \|y\|) \leq g(x)\|x\| + h(y)\|y\|. \tag{3}$$

3. DYNAMIC PARTIAL STATE FEEDBACK

To control the cascade system (1) with time-delay by partial state feedback, we assume that $(z_1, \cdots, z_r)$-dynamics of (1) satisfies the following condition.

Assumption 3.1 For $i = 1, \cdots, r$, there exists a C^1 Lyapunov function $V_{0i}(z_i)$, which is positive definite and proper, such that

$$\frac{\partial V_{0i}}{\partial z_i} f_{0i}(\cdot) \leq -\|z_i\|^2 + \alpha_i(\bar{z}_{i-1}, \bar{z}_{(i-1)d}, \bar{x}_i, \bar{x}_{id}), \tag{4}$$

where $\alpha_i(\bar{z}_{i-1}, \bar{z}_{(i-1)d}, \bar{x}_i, \bar{x}_{id}) \geq 0$ is a C^2 function with $\alpha_i(0,0,0,0) = 0$. □

Theorem 3.2 Under Assumption 3.1, there is a delay-free, dynamic partial state feedback compensator

$$\begin{aligned} \dot{L} &= \theta(L, x), \quad L \in \mathbb{R}^{\mathrm{r}-1}, \\ u &= \beta(L, x), \end{aligned} \tag{5}$$

steering the state $(z_1, \cdots, z_r, x)$ of the time-delay cascade system (1) to the origin while maintaining boundedness of the closed-loop system (1)-(5), where $\theta : \mathbb{R}^{\mathrm{r}-1} \times \mathbb{R}^{\mathrm{r}} \to \mathbb{R}^{\mathrm{r}-1}$ is a continuous function and $\beta : \mathbb{R}^{\mathrm{r}-1} \times \mathbb{R}^{\mathrm{r}} \to \mathbb{R}$ is a smooth function, with $\theta(0,0) = 0$ and $\beta(0,0) = 0$. □

Remark 3.3 In the absence of time-delay, the cascade system (1) was considered in Chen [2009] and Assumption 3.1 reduces to Assumption 2 in Chen [2009]. In the time-delay case, while global stabilization of the cascade system by static partial state feedback may be difficult, it is possible by *dynamic* partial state feedback, as shown by Theorem 3.2. □

Remark 3.4 By hypothesis, $g_i(\cdot)$ is C^1 and vanishes at the origin. This, together with the Taylor expansion and Lemma 2.2, implies the existence of smooth functions $\psi_{ikj}(\cdot) \geq 0$ and $\gamma_{ikj}(\cdot) \geq 0$, $1 \leq j \leq i$, $k = 1, 2$, such that

$$\begin{aligned} |g_i(\bar{z}_i, \bar{z}_{id}, \bar{x}_i, \bar{x}_{id})| \leq \sum_{j=1}^{i} \Big(& \|z_j\|\psi_{i1j}(z_j) + \|z_{jd}\|\psi_{i2j}(z_{jd}) \\ & + |x_j|\gamma_{i1j}(x_j) + |x_{jd}|\gamma_{i2j}(x_{jd}) \Big). \end{aligned} \tag{6}$$

Likewise, by the smoothness and non-negativeness of $\alpha_i(\bar{z}_{i-1}, \bar{z}_{(i-1)d}, \bar{x}_i, \bar{x}_{id})$ with $\alpha_i(0,0,0,0) = 0$, there exist smooth functions $\psi^0_{ikj}(\cdot) \geq 0$, $j = 1, \cdots, i-1$, and $\gamma^0_{ikj}(\cdot) \geq 0$, $j = 1, \cdots, i$, $k = 1, 2$ such that

$$\begin{aligned} &\alpha_i(\bar{z}_{i-1}, \bar{z}_{(i-1)d}, \bar{x}_i, \bar{x}_{id}) \leq \Sigma_{j=1}^{i-1} \Big(\|z_j\|^2 \psi^0_{i1j}(z_j) \\ &+ \|z_{jd}\|^2 \psi^0_{i2j}(z_{jd}) \Big) + \Sigma_{j=1}^{i} \Big(x_j^2 \gamma^0_{i1j}(x_j) + x_{jd}^2 \gamma^0_{i2j}(x_{jd}) \Big) \end{aligned} \tag{7}$$

The inequalities (6) and (7) will be used in the proof of Theorem 3.2. □

Proof of Theorem 3.2: The proof is constructive and based on the idea of updating dynamic gains Zhang and Lin [2014], combined with the tools of changing supply rates Sontag and Teel [1995] and backstepping.

Step 1: For the (z_1, x_1)-subsystem of the time-delay cascade system (1), treat the variable x_2 as a virtual control and let $\xi_1 = x_1$. Then, consider the Lyapunov function $V_1(z_1, x_1, l_1) = \int_0^{V_{01}(z_1)} \eta_1(s)ds + \frac{1}{2}\big(1 + \frac{1}{l_1}\big)\xi_1^2$, where $\eta_1 : [0, +\infty) \to (0, +\infty)$ is a smooth nondecreasing function to be determined later (see (30)) and $l_1(t) \geq 1$ is a dynamic gain defined in (18)-(19).

In view of (4) and (7), it is clear that

$$\begin{aligned} \dot{V}_1 \leq\, & \eta_1(V_{01}(z_1))\big(-\|z_1\|^2 + \xi_1^2\gamma^0_{111}(x_1) + \xi_{1d}^2\gamma^0_{121}(x_{1d}) \big) \\ & + \big(1 + \frac{1}{l_1}\big)\xi_1(x_2 + g_1(z_1, z_{1d}, x_1, x_{1d})) - \frac{\dot{l}_1}{2l_1^2}\xi_1^2. \end{aligned}$$

Using the technique of changing supply rate Sontag and Teel [1995] yields

$$\begin{aligned} &\eta_1(V_{01}(z_1))\big(-\frac{1}{2}\|z_1\|^2 + \xi_1^2\gamma^0_{111}(x_1) + \xi_{1d}^2\gamma^0_{121}(x_{1d}) \big) \\ &\leq \xi_1^2\gamma^{0*}_{111}(x_1) + \xi_{1d}^2\gamma^{0*}_{121}(x_{1d}), \end{aligned}$$

for some smooth functions $\gamma^{0*}_{111}(x_1) \geq 0$ and $\gamma^{0*}_{121}(x_{1d}) \geq 0$.