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Robust Control for State Constrained Uncertain Systems: Attractive Ellipsoid Method Approach

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Abstract: A robust control feedback strategy is developed to solve the stabilization problem of constrained systems with uncertainties and output perturbations. The states are assumed to be constrained inside a given polytope and the perturbations bounded. The control law is developed using an extended version of the attractive ellipsoid method (AEM) approach, and a barrier Lyapunov function (BLF); this is a function whose value goes to infinity whenever its arguments approach to the boundary of a given set. The control parameters are obtained through the solution of some optimization problems related to the approximation of the constraints set and the characterization of a minimal ultimate bounded set for the system trajectories. The implementability of the resulting algorithm is supported by a numerical example and by the comparison with the regular AEM based on a quadratic Lyapunov function.

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1. INTRODUCTION

It is well known that real dynamical systems are affected by noises and that their mathematical models rarely represent accurately, without any kind of uncertainty, actual phenomena. Also, many applications and practical systems are subjected to constraints in the form of physical stoppages, saturation, and safety specifications [Liu and Michel (1994); Sun et al. (1998)] that limit the values of the system states variables. For these reasons, the design of robust control laws ensuring that the closed-loop system solutions do not violate or leave a given set of constraints, even in presence of noisy measurements and system uncertainties, has been of relevance [Kothare et al. (1996); Xu et al. (2015)].

Among the most known results and approaches on the area related to the study and minimization of the effects of perturbations and uncertainties we can consider sliding-mode control, [Shtessel et al. (2014)] which only works mainly for matched perturbations; H_∞ Control [Orlov and Aguilar (2014)], which usually asks for a vanishing condition on the perturbations; and neural networks [Haykin (2009)], which implementability can be difficult. Another well-known approach is the attractive ellipsoid method (AEM), which can deal with unmatched and non-vanishing perturbations, and usually the synthesis of its parameters can be obtained through a linear minimization problem facilitating its implementability.

The characterization of uncertain dynamics by ellipsoidal sets was firstly introduced by the works of [Schweppe (1968)] and [Bertsekas and Rhodes (1971)]. Then, the application of ellipsoids as estimations of sets guaranteed to contain a significant variable was further developed in [Kurzhanskii (1977); Chernousko (1994); Polyak et al. (2004)]. The concept of the asymptotically attractive (invariant) ellipsoid as used in this paper was formalized in [Usoro et al. (1981); Polyak and Topunov (2008)] for linear systems and later extended to nonlinear systems in [Poznyak et al. (2011); Mera et al. (2009)] and [Poznyak et al. (2014)].

The AEM [Gonzalez-Garcia et al. (2009)] is based on the Lyapunov analysis, so it is natural to use other results and methods also based on this analysis to obtain additional features for the closed-loop system, for example using the implicit Lyapunov function method [Polyakov et al. (2014)] to obtain finite time convergence to an ellipsoidal set [Mera et al. (2016)]. With that idea in mind, we decided to use the barrier Lyapunov function (BLF) approach together with the AEM to handle perturbed and constrained systems.

We can name some well-known existing design methods to handle constraints, such as model predictive control [Mayne et al. (2000)], reference governors [Bemporad (1998)], the use of invariant sets [Liu and Michel (1994)], the zeroing Lyapunov function approach [Wieland and Allgöwer (2007)] and the BLF approach [Ngo et al. (2005)]. This last approach consists in using a function whose limit goes to infinity as the system solutions approach to the boundary of a given set, to design the control input. The BLF approach has been widely used with the aid of backstepping, to design control strategies for constrained nonlinear systems [Tee et al. (2009)], tracking of trajectories [Niu and Zhao (2013)], and to implement robust stabilization for applications with constraints [Sane and Bernstein (2002); Ngo et al. (2005)].

Contrasting with the regular approach of using a BLF to design the high-gains inputs directly using the backstepping method, in this paper we use a linear feedback derived from a BLF to estimate and characterize an invariant set for the closed-loop system such that selecting any initial condition in it, it is assured that the system solutions for any $t \geq 0$ do not violate the given constraints. Then, using the AEM we estimate a minimal (in some sense) ultimate bounded region for the

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solutions contained in the invariant set. Using this approach, it is possible to consider constraints on all the states as well as constraints only in some of them.

The structure of this paper is the following. The system description and the formal problem statement are presented in section 2. The main concepts and definitions for the AEM and BLF are introduced in section 3. The main result, a robust linear feedback control strategy, is presented in section 4. Finally, section 6 contains the concluding remarks.

2. PROBLEM STATEMENT

Consider the system

$$\begin{aligned}\dot{x}(t) &= Ax(t)+Bu(t)+\omega(t,x(t)),\\ y(t) &= x(t)+\xi(t),\ \forall t\geq 0,\end{aligned} \tag{1}$$

with $A\in\mathbb{R}^{n\times n}$, and $B\in\mathbb{R}^n$, such that (A,B) is controllable in Kalman sense, whose solutions are constrained in $\mathbb{R}^n$ inside a polytope $\mathscr{P}:=\{x: a_i^T x\leq 1, i=1,\ldots,k\}$, where $a_i\in\mathbb{R}^n$ is a set of given vectors, $x\in\mathbb{R}^n$ is the state vector, $u\in\mathbb{R}$ is the control input, $y\in\mathbb{R}^n$ is the measurable output, $\omega:\mathbb{R}\times\mathbb{R}^n\to\mathbb{R}^n$ describes exogenous vanishing disturbances and uncertainties (e.g. uncertain nonlinearities of the system), such that for the symmetric matrices $Q_\omega, Q_x\in\mathbb{R}^{n\times n}$ the next inequality (i.e vanishing condition) is fulfilled

$$\omega^T(t)Q_\omega\omega(t)\leq x^T(t)Q_x x(t),\ \forall t\geq 0, \tag{2}$$

and $\xi\in\mathbb{R}^n$ is an unknown but bounded and locally measurable perturbation, which is bounded for the symmetric matrix $Q_\xi\in\mathbb{R}^{n\times n}$ as

$$\xi^T(t)Q_\xi\xi(t)\leq 1,\ \forall t\geq 0. \tag{3}$$

The objective of this paper is to design a robust feedback control strategy for system (1), such that the closed loop system solutions remain inside the constraints set for all $t\geq 0$, and additionally converge asymptotically to a minimal size (in a certain sense) ellipsoid contained in it, despite having noisy measurements and uncertainties. A BLF and the AEM are used to device the control strategy and obtain the corresponding design parameters.

3. PRELIMINARIES

Considering the system of the form

$$\dot{x}(t)=f(x(t),\xi(t)),\ \forall t\geq 0,\ x(0)=x_0,\ \xi(0)=\xi_0; \tag{4}$$

where $x(t)\in\mathbb{R}^n$ is the state vector, $\xi(t)\in\mathbb{R}^n$ is an unknown but bounded perturbation

$$\|\xi(t)\|\leq l_0,\ \forall t\geq 0,\ l_0\in\mathbb{R}_+,$$

and $f:\mathbb{R}^n\times\mathbb{R}^n\to\mathbb{R}^n$ is a nonlinear continuous function. Also, consider that the states are constrained in $\mathbb{R}^n$ by the polytope

$$\mathscr{P}:=\{x: a_i^T x\leq 1, i=1,\ldots,k\}, \tag{5}$$

where $a_i\in\mathbb{R}^n$ are given vectors, and assume that the ellipsoid

$$\mathscr{E}_x:=\left\{x\in\mathbb{R}^n: x^T\tilde{P}x\leq 1\right\},\quad \tilde{P}\in\mathbb{R}^{n\times n},\ \tilde{P}=\tilde{P}^T>0. \tag{6}$$

contained in $\mathscr{P}$ is an invariant set of (4). Additionally, we consider the notation, for any $\theta\in\mathbb{R}^n$

$$\|\theta\|_{\mathscr{E}_x}:=\inf_{\eta\in\mathscr{E}_x}\|\theta-\eta\|,$$

as the distance from a point θ to a set $\mathscr{E}_x$.

The idea behind the BLF approach is to use a function which limit value goes to infinity whenever its arguments approach the boundary of some set as a Lyapunov function to analyze the system stability.

Definition 1. (Barrier Lyapunov Function). Let the set $\mathscr{D}\subset\mathbb{R}^n$ be an open set with the boundary $\partial\mathscr{D}$, assume that there exists an invariant set for (4) contained in $\mathscr{D}$ and let $V:\mathscr{D}\to\mathbb{R}_+$ be a continuous function in $\mathbb{R}^n$. V is a BLF if it is positive definite, continuously differentiable in $\mathscr{D}$,

$$\lim_{x\to\partial\mathscr{D}^-} V(x)\to+\infty,$$

and $V(x)\leq b$, $\forall t\geq 0$ for some $b\in\mathbb{R}_+$ for any $x(0)\in\mathscr{D}$.

If $\dot{V}\leq 0$ and $x(0)\in\mathscr{D}$ it is clear that $b=V(x(0))$, and that any future trajectory is bounded inside $\mathscr{D}$. However, it is desirable for many applications that the trajectories not only remain inside a given set but converge to a small (in a given sense) neighborhood of the origin. To characterize this minimal region formally we present the definition from [Poznyak et al. (2014)] of the Attractive Ellipsoid.

Definition 2. (Asymptotically Attractive Ellipsoid). The set $\mathscr{E}_x$, is an asymptotically attractive ellipsoid for the system (4) if $\|x(t,x_0)\|_{\mathscr{E}_x}\to 0$, as $t\to\infty$, for any $x_0\in\mathbb{R}^n$.

Considering the concepts presented in this section, the idea developed in the next section is to design a control input that ensures that the closed-loop system trajectories starting in the invariant set $\mathscr{D}$ converge asymptotically to a "minimal" attractive ellipsoid $\mathscr{E}_x\subset\mathscr{D}$.

4. CONTROL DESIGN

In order to reduce the complexity of the stability analysis and the control design, the constraints set (5) can be approximated by an ellipsoidal set completely contained in it. It is not difficult to see that there exists a family of ellipsoids

$$\mathscr{R}:=\{x: x^T Rx\leq 1\}, \tag{7}$$

parametrized by a positive definite matrix $R\in\mathbb{R}^{n\times n}$, contained in $\mathscr{P}$ if

$$a_i^T R^{-1} a_i\leq 1,\ \forall\, a_i. \tag{8}$$

Remark 1. For the stability analysis we can chose the origin of the system (4), the center of the ellipsoid $\mathscr{R}$ and the barycenter of the polytope $\mathscr{P}$ as the exact same point without any lose of generality. However, the inequalities (8) are also valid if the origin is required to be included in the polytope but to be different from its barycenter.

Let us consider the following BLF candidate

$$V(x)=\log\left(\frac{1}{1-x^T Rx}\right), \tag{9}$$

where $log(\cdot)$ is the natural logarithmic function. For any $x(0)\in\mathbb{R}^n$, note that $V(x)\to+\infty$ as x tends to the boundary of $\mathscr{R}$ from the inside, and if $\dot{V}\leq 0$ then $\mathscr{R}$ is the set $\mathscr{D}$ from definition 1.

Theorem 1. Let $\gamma,\beta\in\mathbb{R}_+$, $a_i\in\mathbb{R}^n$, $i=1,2,\ldots,k$, a set of given vectors that define a polytope $\mathscr{P}$ in $\mathbb{R}^n$, the matrices A and B defined as in (1), the matrices

$$W_1:=\begin{pmatrix} \begin{matrix}AX_1+X_1A^T+BY\\ +Y^TB^T+\gamma X_2+\beta X_1\end{matrix} & \begin{matrix}X_1A^T+Y^TB^T\\ +X_1-X_2\end{matrix} & BY & I_{n\times n}\\ AX_1+BY+X_1-X_2 & -2X_2 & BY & I_{n\times n}\\ Y^TB^T & Y^TB^T & -\gamma X_1 & 0\\ I_{n\times n} & I_{n\times n} & 0 & -\gamma Q_\omega \end{pmatrix},$$

$$W_2:=\begin{pmatrix}\beta X_1 & \gamma X_1\\ \gamma X_1 & \gamma Q_x^{-1}\end{pmatrix},$$

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