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Robust Soft-Landing Control with Quantized Input

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Abstract: We propose a controller architecture for soft-landing control with quantized input. The objective of the soft-landing problem is to achieve precise positioning of a moving object at a target position, while ensuring the velocity decreases as the target is approached. In this paper, we formulate the soft-landing problem as a constrained control problem. Our approach combines traditional convex model predictive control with a rounding rule that quantizes the input. The rounding rule is designed to minimize the error between the requested and quantized inputs. A robust control invariant set is used to ensure that the rounding errors do not lead to constraint violations. We demonstrate our approach for a transportation system case study.

Keywords: Robust Model Predictive Control, Invariant Sets, Quantized Control

1. INTRODUCTION

Many control applications in the automotive, aerospace, manufacturing, and transportation fields require the precise positioning of a moving object at a desired location while ensuring the the velocity of the object decreases as the target is approached. The resulting soft-landing (also called soft-contact) avoids damage and reduces wear. One example of soft-landing is the control of valves in camless engines, which requires the high-speed closing of a valve in its seating while avoiding rough impacts that reduce component operating life (see Hoffmann et al. (2003)). Another example is docking of spacecraft which requires the docking spacecraft to make soft-contact to avoid the spacecraft ricocheting or suffering damage (see Weiss et al. (2012)). Soft-landing is also important for rider comfort during the automatic stopping of vehicles (see Bu and Tan (2007)).

The soft-landing problem can be formulated as a constrained control problem where constraints are placed on the object velocity relative to its position so that the object slows as it approaches the target position. Hence the soft-landing problem can be solved using constrained control techniques. An early approach to the soft-landing problem was based on reference governors (see Kolmanovsky and Gilbert (2001)). This approach can guarantee constraint satisfaction but has limited performance since reference governors can only manipulate the reference of a linearly pre-compensated system. More recently, model predictive control (MPC) has been used to solve the soft-landing problem (see Di Cairano et al. (2014)). In Di Cairano et al. (2007) soft-landing MPC was applied to soft-landing for valves in camless engines. In Di Cairano et al. (2012) and Weiss et al. (2012) soft-landing MPC was applied to soft-landing for space-craft docking.

In this paper, we consider the soft-landing problem when the input is restricted to a finite set. Model predictive control has been applied to system with finite input in the literature. Aguilera and Quevedo (2011) studied the stabilization of systems with a finite number of inputs using model predictive control. Corona et al. (2006) focused on the optimality of model predictive control for finite input system. In the soft-landing problem, our main concern is with guaranteeing constraint satisfaction rather than optimality or stability. Picasso et al. (2002) presented a method for computing control invariant sets for linear systems with finite input. For numerical simplicity, we adopt the rounding rule approach from Kirches (2011). Our approach combines a convex model predictive controller which guarantees robust state constraint satisfaction and a rounding rule that ensures the input lies in the finite set. Our rounding rule is designed using Voronoi partitions. This approach has been previously used in Bullo and Liberzon (2006). The convex MPC and rounding rule are designed jointly to ensure robust constraint satisfaction.

This paper is organized as follows. In Section 2 we formally define the dynamics, constraints, and control objectives for the soft-landing problem. In Section 3 we describe our control algorithm for solving the soft-landing problem. We pay particular attention to the issue of ensuring our controller is robust to model uncertainty and rounding errors. In Section 4 we demonstrate our control algorithm on a transportation system case study.

2. SOFT-LANDING PROBLEM

In this section we define the dynamics, constraints, and control objectives of the soft-landing problem.

2.1 Soft-Landing Dynamics

We consider an inertial object moving in a one-dimensional space described by the dynamics

$$\dot{x}_m(t) = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & -\frac{b}{m} \end{bmatrix}}_{\bar{A}_m} x_m(t) + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}}_{\bar{B}_m} q_f(t) \tag{1}$$

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where the state $x_m(t) = [y(t), \dot{y}(t)]^T \in \mathbb{R}^2$ of the systems is the position $y(t)$ and velocity $\dot{y}(t)$ of the inertial object, m is the mass of the object, b is the viscous friction coefficient, and $q_f(t) \in \mathbb{R}^1$ is the controlled force on the object.

The controlled force $q_f(t)$ is the output of a linear filter that captures the dynamics of the actuator. The filter is described by the state-space model

$$\dot{x}_q(t) = \bar{A}_q x_q(t) + \bar{B}_q q(t) \tag{2a}$$

$$q_f(t) = \bar{C}_q x_q(t) + \bar{D}_q q(t) \tag{2b}$$

where $x_q \in \mathbb{R}^{n_q}$ is the state of the input filter and $q(t) \in \mathbb{R}^m$ is the input command. The soft-landing problem assumes specific dynamics and constraints for the inertial subsystem, but the input dynamics can be arbitrary and are unconstrained. The soft-landing problem can also include a disturbance force d. In addition the controlled input force $q_f(t)$ can depend on the state $x_m(t)$ of the inertia systems. However, for simplicity, we do not consider these cases in this paper.

The state of the composite system $x(t) = [x_m(t), x_q(t)]$ is the state of the inertial system $x_m(t)$ and input filter $x_q(t)$. In discrete-time, the dynamics of the composite system are modeled by

$$\begin{bmatrix} x_m(k+1) \\ x_q(k+1) \end{bmatrix} = \underbrace{\begin{bmatrix} \hat{A}_m & \hat{B}_m \hat{C}_q \\ 0 & \hat{A}_q \end{bmatrix}}_{A} \begin{bmatrix} x_m(k) \\ x_q(k) \end{bmatrix} + \underbrace{\begin{bmatrix} \hat{B}_m \hat{D}_q \\ \hat{B}_q \end{bmatrix}}_{B} q(k) \tag{3}$$

where $(\hat{A}_m, \hat{B}_m)$ and $(\hat{A}_q, \hat{B}_q, \hat{C}_q, \hat{D}_q)$ are the discrete-time transformations of $(\bar{A}_m, \bar{B}_m)$ and $(\bar{A}_q, \bar{B}_q, \bar{C}_q, \bar{D}_q)$ respectively. We use the short-hand $x(k) = x(t_k)$ and $q(k) = q(t_k)$ to denote the state and input, respectively, at time $t_k = t_0 + k\Delta t$ where $k \in \mathbb{N}$ and Δt is the discrete-time sample period.

2.2 Soft-Landing Constraints

In this section we describe the state and input constraints for the soft-landing problem. The state constraints only apply to the state $x_m(t)$ of the inertial system.

The objective of the soft-landing problem is to bring the inertial object to a stop in a neighborhood of the origin, called the target set

$$\mathcal{T} = \left\{ \begin{bmatrix} x_{m,1} \\ x_{m,2} \end{bmatrix} : \begin{array}{c} x_{min} \leq x_{m,1} \leq x_{max} \\ x_{m,2} = 0 \end{array} \right\} \subseteq \mathbb{R}^2$$

where $x_{min} < 0 < x_{max}$ are the maximum deviations of the position $x_{m,1}$ from the origin. We only require that the state $x_m(t)$ reaches the target set, not that it remains in the target set. This is motivated by several practical applications in which, upon entering the target set, the dynamics change in such a way to keep the state in the target set. For instance static friction is used to hold valves in their seating, clamps are used hold spacecraft in place during docking, and parking brakes are used to keep elevators at floor level.

We do not want the state $x_m(k)$ of the inertial systems to approach the target set $\mathcal{T}$ with a velocity that is too fast or too slow. Therefore we introduce the "soft-landing" cone constraint to control the approach velocity

$$\mathcal{S} = \left\{ \begin{bmatrix} x_{m,1} \\ x_{m,2} \end{bmatrix} : \begin{array}{c} x_{m,2} + \gamma_{max}(x_{m,1} - x_{max}) \leq 0 \\ x_{m,2} + \gamma_{min}(x_{m,1} - x_{min}) \geq 0 \end{array} \right\} \tag{4}$$

where $\gamma_{max}, \gamma_{min} \in \mathbb{R}_+$ with $\gamma_{min} < \gamma_{max}$ are spatial deceleration coefficients. This constraint bounds the velocity $x_{m,2}(k) = \dot{y}(t_k)$ of the inertial system as a function of position $x_{m,1}(k) = y(t_k)$ to ensures the velocity decreases smoothly as the inertial system approaches the target set $\mathcal{T} \subseteq \mathbb{R}^2$. The state constraint set is given by the unbounded polytope

$$\mathcal{X} = \mathcal{S} \times \mathbb{R}^{n_q} \subset \mathbb{R}^n \tag{5}$$

where $n = 2 + n_q$ is the dimension of the composite system (3). In the soft-landing problem there are no constraints on the state of the input $x_q \in \mathbb{R}^{n_q}$ filter.

The nonlinearity of this problem is due to the quantization of the input $q(k)$ which is drawn from a finite set $\mathcal{Q} \subset \mathbb{R}^m$ where $|\mathcal{Q}| < \infty$. We assume that the convex-hull $\text{conv}(\mathcal{Q})$ of the input set $\mathcal{Q}$ contains the origin in its interior $0 \in \text{conv}(\mathcal{Q})$.

2.3 Soft-Landing Objectives

The objective of the soft-landing problem is to generate an input trajectory that drives the inertial object to the target set while satisfying state constraints. The soft-landing problem is formally stated below.

Problem 1. (Soft-landing). Select a feasible input trajectory $q(k) \in \mathcal{Q}$ for $k \in \mathbb{N}$ such that the state trajectory $x(k) = [x_m(k), x_q(k)]^T$ resulting from the dynamics (3) satisfies the state constraints $x(k) \in \mathcal{X}$ for all $k \in \mathbb{N}$ and converges $x(k) \to \mathcal{T}$ to the target set $\mathcal{T}$ i.e. there exists $f \in \mathbb{N}$ such that $x(f) \in \mathcal{T}$.

If the final time $f = \infty$ is infinite, then we mean that the state asymptotically converges the target set $x_m(k) \to \mathcal{T}$.

In Di Cairano et al. (2014) it was shown that a feasible state trajectory $x(k) \in \mathcal{X}$ that satisfies the dynamics (3) will necessarily converge to the target set. Thus Problem 1 can be solved by simply finding a feasible input trajectory $q(k) \in \mathcal{Q}$ that produces a persistently feasible state trajectory $x(k) \in \mathcal{X}$. In Di Cairano et al. (2014), Problem 1 was solved using convex model predictive control with a robust control invariant set that guaranteed persistent feasibility. In this paper we extend this result to the case where the input is quantized.

3. SOFT-LANDING CONTROL DESIGN

In this section we describe our controller for solving the Soft-Landing Problem 1. Our controller consists of two parts connected in series: a convex model predictive controller and a rounding rule. The convex model predictive controller is used to ensure robust state constraint satisfaction. The rounding rule is used to ensure satisfaction of the input constraint. The rounding rule will be described in Section 3.2 and the model predictive controller will be described in Section 3.3.

3.1 Model Uncertainty

The mass m and the viscous friction coefficient b of the inertial system are uncertain and thus the matrices $\bar{A}_m$ and $\bar{B}_m$ are uncertain. In addition, the dynamics matrices $(\bar{A}_q, \bar{B}_q, \bar{C}_q, \bar{D}_q)$ of the input dynamics (2) may