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Sampled-data, output feedback predictive control of uncertain, nonlinear systems

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Abstract: We present a model predictive control approach for uncertain, continuous-time, constrained, nonlinear systems with noisy, discrete time measurements. The proposed discrete-time controller combines a state estimator with a suitably robustified predictive control law. For this purpose the continuous-time dynamics is discretized and rigorous bounds on the truncation errors as well as the noise are derived. We combine these bounds with ideas from tube based predictive control to derive conditions to guarantee constraint satisfaction, robust recursive feasibility and robust stability. An example illustrates the results.

1. INTRODUCTION

Model predictive control (MPC) is frequently used to control nonlinear systems with constraints, see (Lucia et al., 2016; Mayne, 2014; Rawlings and Mayne, 2009). MPC uses at each time instant the first part of the solution of an optimal control problem as feedback. Unfortunately, for real systems usually the model of the continuous-time system is uncertain, the state needs to be reconstructed from noisy measurements and the digital control hardware works in discrete-time. These challenges need to be taken into account to avoid infeasibility, constraint violation and instability of the closed loop system.

This work considers the robust, output feedback control of continuous-time, uncertain, nonlinear systems by a discrete-time controller. Similar to the state feedback case (investigated in (Kögel and Findeisen, 2015a)): we first discretize the system using a numerical integration scheme for which the truncation error and the effect of the process noise can be explicitly bounded. Therefore, we compute a set of 'disturbances' such that the uncertain continuous-time behavior is given by the nominal discrete-time system plus an additive 'disturbance' from this set, i.e. the continuous-time behavior is included in the discrete-time behavior. In a second step, a discrete-time predictive output feedback controller is designed, which is robust with respect to the process and measurement noise as well as the discretization error.

By now many works address robustness/robust design of MPC, see e.g. (Rawlings and Mayne, 2009; Streif et al., 2014) for an overview. Inherent robustness of output feedback MPC is investigated in e.g. (Imsland et al., 2003). Different approaches for robust, output-feedback MPC based on robust optimization (see e.g. (Copp and Hespanha, 2014)), or tube MPC (see e.g. (Kögel and Findeisen, 2015a,b,c; Mayne et al., 2006, 2009)), passivity plus dissipativity (Yu et al., 2013) or Roset et al. (2008) input-to-state stability have been proposed.

We use ideas similar to those of (Kögel and Findeisen, 2015a,b,c; Mayne et al., 2006, 2009) to guarantee robust constraint satisfaction and robust stability of the discrete-time system, which hold due to the employed, consist discretization also for the real closed loop system.

The remainder is structured as follows. Section 2 states the problem. Section 3 considers the consistent discretization of the dynamics. Section 4 deals with state estimation. Section 5 investigates robustness properties of the closed loop system. Section 6 illustrates the results by an example.

Notation: For a compact set $\mathbb{M}$, $\|\mathbb{M}\|_\infty = \max_{x\in\mathbb{M}} \|x\|_\infty$. For two sets $A \oplus B/A \ominus B$ denotes the Minkowski sum/difference, see e.g. (Rawlings and Mayne, 2009). For a vector x, $x_{[i]}$ is the ith component of x. $\star$ denotes optimal values of variables/functions.

2. PROBLEM FORMULATION

This section outlines the considered class of systems and sketches the proposed output feedback consisting of a robust model predictive control law and a state estimator.

2.1 System class

Considered are nonlinear, continuous-time systems affected by bounded, additive disturbances of the form

$$\begin{aligned} \dot{x}(t) =& f(x(t), u(t), w(t)), && \text{(1a)}\\ =& Ax(t) + Bu(t) + w(t) + g(x(t), u(t)), &&\\ & w(t) \in \mathbb{W}, && \text{(1b)} \end{aligned}$$

where x is the system state, u the applied input, w an unknown, piece-wise continuous disturbance, also called process noise, and $t \geq 0$ is the time. Disturbances are drawn from the set $\mathbb{W} \subset \mathbb{R}^n$, which contains the origin and is assumed to be convex and compact. We assume that the nonlinearity $g(x(t), u(t))$ is locally Lipschitz continuous in $x(t)$ and $u(t)$ and satisfies $g(0, 0) = 0$, i.e. the origin is a steady state for the nondisturbed system.

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The input $u(t)$ and system state $x(t)$ need to satisfy

$$x(t) \in \mathbb{X},\ u(t) \in \mathbb{U},\ t \geq 0, \tag{2}$$

where the sets $\mathbb{U} \subset \mathbb{R}^p$, $\mathbb{X} \subset \mathbb{R}^n$ are assumed to be compact, convex and contain a neighborhood of the origins.

The system state x is not directly available and needs to be estimated from noisy measurements y. The measurements y are taken at the sampling instants $t_k = kT$,[1] where $T > 0$ is the sampling time. The measurements satisfy

$$y_k = Cx_k + v_k,\ v_k \in \mathbb{V}, \tag{3}$$

where v_k denotes the measurement noise, which is restricted to the compact, convex set $\mathbb{V} \subset \mathbb{R}^q$. Moreover, an estimate $\hat{x}_0$ of the initial state $x(0)$ and a compact, convex set $\mathbb{E}_0$ restricting the initial estimation error $e_0 = x(0) - \hat{x}_0$ to $e_0 \in \mathbb{E}_0$ are available.

The objective is to design an output feedback controller for the system (1)-(3) such that the closed loop system is robustly stable and satisfies the constraints (2) for every admissible noise realization. This controller is evaluated periodically at the sampling instants t_k, when new measurements are available. For simplicity we assume that the input $u(t)$ is held constant between the sampling instants:

$$u(t) = u_k,\ \text{ for } t \in [t_k, t_{k+1}). \tag{4}$$

2.2 Proposed approach

The basic idea is to first discretize the continuous-time dynamics and to bound the effects of the truncation and the process noise. Second, based on these bounds a robust predictive output feedback controller is designed for the discrete-time approximation, such that the overall closed loop system is robust with respect to the process and measurement noise as well as the truncation error. In the remainder of this section, we sketch the approximate discretization and the proposed control framework combining a state estimator with a robust, predictive control law.

Approximate discretization with guarantees
Using the input parametrization (4) the state x_{k+1} is given by the state x_k at the current sampling time, the applied input u_k and the realization of the disturbance w between t_k and t_{k+1}. In the nominal case ($w(t) \equiv 0$) this relation is a map from x_k, u_k to x_{k+1} given by the solution of a differential equation:

$$\mathcal{F}(x,u) = \xi(T),\ \text{ where } \dot{\xi}(t) = f(\xi(t), u_k, 0),\ \xi(0) = x_k. \tag{5}$$

Similarly, we define the set of reachable states x_{k+1} from the state x_k with the input u_k in the uncertain case:

Definition 1. (Reachable set)
The reachable set $\mathcal{H}(x,u)$ for (1), (4) *is given by the states reachable in T time units from x using the dynamics* (1), *a constant input u and any admissible realization of $w(t)$.*

Clearly, $\mathcal{H}(x,u)$ is given by all $\xi(T)$ satisfying

$$\dot{\xi}(t) = f(\xi(t), u, w(t)),\ w(t) \in \mathbb{W},\ t \in [0,T],\ \xi(0) = x.$$

Unfortunately, one can determine an explicit expression for the reachable set $\mathcal{H}(x,u)$ or just for the exact discretization $\mathcal{F}(x,u)$ for nonlinear systems only in rare special cases, see e.g. (Khalil, 2002; Rihm, 1994). To still provide guarantees the idea is to approximately discretize (1), (4) and bound the arising truncation error as well as the influence of the process noise $w(t)$ similar as in (Kögel and Findeisen, 2015a).

The key idea used in this work is to replace (1), (4) by

$$x_{k+1} = F(x_k, u_k) + d_k,\ d_k \in \mathbb{D}(x_k, u_k), \tag{6}$$

where the map $\mathcal{F}(x,u)$ is replaced by an approximation $F(x,u)$. In this work we limit our attention to the Euler method with a single step of length T for a time interval:

$$F(x,u) = x + Tf(x,u,0). \tag{7}$$

The set $\mathbb{D}(x_k, u_k)$ is chosen such that consistency is guaranteed:

$$\mathcal{H}(x,u) \subseteq \underbrace{\{F(x,u)\} \oplus \mathbb{D}(x,u),\ \forall x \in \mathbb{X}, \forall u \in \mathbb{U}}_{=H(x,u)}. \tag{8}$$

We refer to the following section for more details.

State estimator
The state estimator obtains an estimate $\hat{x}_k$ of x_k. We focus on estimators with the following structure

$$\hat{x}_{k+1} = F(\hat{x}_k, u_k) + \mathcal{L}(\hat{x}_k, y_k), \tag{9}$$

where the correction term $\mathcal{L}(\hat{x}_k, y_k)$ improves/stabilizes the estimation error. Details are given in Section 4.

Predictive output feedback scheme
The proposed robust MPC scheme utilizes the state estimate $\hat{x}_k$ and the approximate discretization to obtain input and state sequences[2]

$$\mathbf{x}_k = \left(x'_{k|k} \ \dots \ x'_{k+N|k}\right)',\ \mathbf{u}_k = \left(u'_{k|k} \ \dots \ u'_{k+N-1|k}\right)',$$

defined over a horizon N. $\mathbf{x}_k$, $\mathbf{u}_k$ need to be consistent with the estimate $\hat{x}_k$, and the (approximate) dynamics:

$$x_{i+1+k|k} = F(x_{i+k|k}, u_{i+k|k}),\ i = 0, \dots, N-1, \tag{10a}$$

$$x_{k|k} = \hat{x}_k. \tag{10b}$$

Moreover, the sequences $\mathbf{x}_k$ and $\mathbf{u}_k$ need to satisfy constraints of the form (as defined below)

$$\mathbf{u}_k \in \tilde{\mathbf{U}}, \qquad \mathbf{x}_k \in \tilde{\mathbf{X}}, \tag{11}$$

and need to minimize the nominal performance index

$$J(\mathbf{x}_k, \mathbf{u}_k) = \sum_{j=k}^{N+k-1} \ell(x_{j|k}, u_{j|k}) + P(x_{k+N|k}),$$

given by a terminal cost $P(x)$ and a stage cost $\ell(x,u)$.

To sum up: the control input is determined at t_k by the solution of the optimization problem $\mathcal{O}(\hat{x}_k)$

$$\mathcal{O}(\hat{x}_k): \min_{\mathbf{x}_k, \mathbf{u}_k} J(\mathbf{x}_k, \mathbf{u}_k) \text{ s.t. (10), (11).} \tag{12a}$$

This problem maps the state estimate $\hat{x}_k$ into a (possible non-unique) optimal state trajectory $\mathbf{x}_k^\star$ and input sequence $\mathbf{u}_k^\star$. The first optimized input is the feedback

$$u_k = u^\star_{k|k}. \tag{12b}$$

In Section 5 we present conditions such that the considered output feedback combining (9) and (12) guarantees robust stability and robust constraint satisfaction despite the fact that the uncertainties are not explicitly considered within (12), by choosing the constraints (11) appropriately.

[1] In the following we write x_k to denote $x(t_k)$.

[2] $i + k|k$ denotes a prediction i steps into the future.