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Dropout feedback parametrized policies for stochastic predictive controller

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Abstract: This article presents a novel control policy class for the networked control of linear dynamical systems with bounded inputs. The control channel is assumed to have i.i.d. Bernoulli packet dropouts. Our proposed class of policies is parametrized relative to the past dropouts. We show how to augment the underlying optimization problem with a constant negative drift constraint in order to ensure mean-square boundedness of the closed-loop states. The resulting convex quadratic program can be solved periodically online. The states of the closed loop plant under the receding horizon implementation of the proposed class of policies are mean square bounded for any positive bound on the control.

Keywords: Erasure channel, Stochastic Model Predictive Control, Networked system, Multiplicative noise, Unreliable channel.

1. INTRODUCTION

Stochastic Model Predictive Control (SMPC) has the capability to handle constraints while minimizing some average performance index under the effects of uncertainties. Because of its ability to handle constraints, MPC is highly utilized in networked systems as well. In a broad sense, SMPC approaches can be categorized into three classes: the stochastic tube approach (Cannon et al., 2011), the scenario based approach (Campi et al., 2009), and the affine disturbance feedback approach (Oldewurtel et al., 2008; Hokayem et al., 2009). It is well known that the feedback of past additive disturbances leads to convex problems, whereas [1] state feedback approach to non-convexity of the set of decision variables (Goulart et al., 2006). In this article we consider deterministic linear systems controlled over a noisy communication channel that randomly severs the control signals. We demonstrate that the parametrization relative to past dropouts also leads to convex problems.

The theoretical development of the deterministic MPC has been matured over the years, but stochastic receding horizon control still lacks a comprehensive and unified treatment (Mayne, 2014). In order to achieve a convex set of feasible decision variables, a disturbance feedback approach has been proposed (Garstka and Wets, 1974; Goulart et al., 2006) and further studied with bounded disturbances (Löfberg, 2003) and Gaussian disturbances (Van Hessem and Bosgra, 2002). To satisfy hard bounds on the control, (Hokayem et al., 2009) used saturated values of past disturbances. This saturated disturbance feedback policy is applied to networked systems with enough control authority (Chatterjee et al., 2010) and also for any positive bound on the control in our recent work (Mishra et al., 2016).

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[1] optimization over open loop input sequences is not optimal (Kumar and Varaiya, 1986)

The affine disturbance feedback approach under unreliable uplink has three important ingredients : the class of control policies, the stability constraints, and the transmission protocol. We shall discuss all three in this article. The control policy class enables us to take the past dropouts as feedback, and is compatible with the stability constraints. Stability constraints allow us to transcend beyond the regime of terminal set method. In order to ensure recursive feasibility and mean square boundedness *for any positive bound on control*, the stability constraints presented in this article are different from those in (Hokayem et al., 2010). The parameters of the control policy can be transmitted in several different ways through the control channel and the scheme of the transmission of the parameters affects the optimization problem. Therefore, the transmission protocol should also be considered at the synthesis stage. We can easily introduce the effect of transmission protocol on the optimization problem with help of the proposed control policy class. For the demonstration purpose, we consider a simple transmission protocol in which computation or storage facilities at the actuator end are not required.

The article is organized as follows: In §2 we establish the notation and definitions of the plant and its properties. In §3 we provide the main result. The proof of our result is documented in a consolidated fashion in the appendix.

We let $\mathbb{R}$ denote the real numbers and $\mathbb{N}$ denote the natural numbers. The standard Schur products of matrices is denoted by $\odot$. The notation $\mathbb{E}_z[\cdot]$ stands for the conditional expectation with given initial condition z. We denote by $s_{n:k}$ the vector $\left(s_n^\top \; s_{n+1}^\top \; \cdots \; s_{n+k-1}^\top\right)^\top$, $k \in \mathbb{N}$, for any sequence $(s_n)_{n\in\mathbb{N}}$ taking values in some Euclidean space. The i^{th} component of a given vector V is denoted by $V^{(i)}$. Similarly, $M^{(i,:)}$ and $M^{(:,i)}$ denote the i^{th} row and i^{th} column of a given matrix M, respectively. The vector of all 1's of length k is denoted by $\mathbb{1}_{1:k}$.

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2. PROBLEM SETUP

2.1 Dynamics and objective function

Consider the linear time-invariant control system with additive process noise and controlled over an erasure channel given by

(1) $$x_{t+1} = Ax_t + B\nu_t u_t, \quad x_0 = \overline{x},$$

where

((1)-a) $x_t \in \mathbb{R}^d$, $A \in \mathbb{R}^{d\times d}, B \in \mathbb{R}^{d\times m}$ are given matrices, $\overline{x} \in \mathbb{R}^d$ is a given vector, the control u_t at time t takes values in the set

(2) $$\mathbb{U} := \{v \in \mathbb{R}^m \mid \|v\|_\infty \leqslant U_{\max}\},$$

((1)-b) $(\nu_t)_{t\in\mathbb{N}}$ is a sequence of i.i.d. Bernoulli $\{0,1\}$ random variables, and

((1)-c) the state x_t is measured perfectly at each time t.

A control policy π is a sequence $(\pi_0,\ldots,\pi_t,\ldots)$ of Borel measurable maps $\pi_t : \mathbb{R}^d \longrightarrow \mathbb{U}$. Policies of finite length $(\pi_t,\pi_{t+1},\ldots,\pi_{t+T-1})$ for some $T \in \mathbb{N}$ will be denoted by $\pi_{t:T}$ in the sequel.

Let symmetric and non-negative definite matrices $Q, Q_f \in \mathbb{R}^{d\times d}$ and a symmetric and positive definite matrix $R \in \mathbb{R}^{m\times m}$ be given, and define the cost-per-stage function $c_s : \mathbb{R}^d \times \mathbb{U} \longrightarrow [0,+\infty[$ and the final cost function $c_f : \mathbb{R}^d \longrightarrow [0,+\infty[$ by

$$c_s(z,v) := \langle z, Qz\rangle + \langle v, Rv\rangle \text{ and } c_f(z) = \langle z, Q_f z\rangle,$$

respectively. Fix an optimization horizon $T \in \mathbb{N}$, and consider the objective function at time t given the state x_t:

(3) $$V_t := \mathbb{E}_{x_t}\left[\sum_{k=0}^{T-1} c_s(x_{t+k}, u^a_{t+k}) + c_f(x_{t+T})\right],$$

where $u^a_t = \nu_t u_t$ is available control at the actuator end at time t. The cost function V_t, therefore considers the control effort that occurs at the actuator end, not the computed control, see Fig. 1. At each time instant t we are interested in minimizing the objective function V_t over the class of causal history-dependent feedback strategies Π defined by

$$u_{t+\ell} = \pi_{t+\ell}(x_t, x_{t+1},\ldots,x_{t+\ell})$$

while satisfying $u_t \in \mathbb{U}$ for each t.

The *receding horizon control strategy* for a given control horizon $N_c \in \{1,\ldots,T\}$ and time t consists of successive application of the following steps:

(i) measure the state x_t and determine an admissible optimal feedback policy $\pi^\star_{t:N} \in \Pi$ that minimizes V_t at time t,
(ii) apply the first N_c elements $\pi^\star_{t:N_c-1}$ of this policy,
(iii) increase t to $t+N_c$ and return to step (i).

The states, controls and dropouts over one horizon T admit the following description under *unreliable control channel*, in the sense that solely the control action u_t is sent over the communication channel at each time t:

(4) $$x_{t:T+1} = \mathcal{A}x_t + \mathcal{B}u^a_{t:T},$$

where

$$u^a_{t:T} := \begin{pmatrix} \nu_t u_t \\ \nu_{t+1}u_{t+1} \\ \vdots \\ \nu_{t+T-1}u_{t+T-1}\end{pmatrix},$$

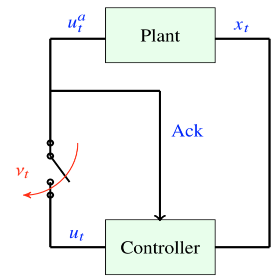

Fig. 1. The acknowledgment of successful transmission of the control signal is causally available to the controller.

$$\mathcal{A} := \begin{pmatrix} I_d \\ A \\ \vdots \\ A^T\end{pmatrix}, \mathcal{B} := \begin{pmatrix} 0_{d\times m} & 0_{d\times m} & \cdots & 0_{d\times m} \\ B & 0_{d\times m} & \cdots & 0_{d\times m} \\ \vdots & \vdots & \ddots & \vdots \\ A^{T-1}B & A^{T-2}B & \ldots & B\end{pmatrix}, \text{ and } \mathcal{D} := \begin{pmatrix} 0_{d\times d} & 0_{d\times d} & \cdots & 0_{d\times d} \\ I_d & 0_{d\times d} & \cdots & 0_{d\times d} \\ \vdots & \vdots & \ddots & \vdots \\ A^{T-1} & A^{T-2} & \ldots & I_d\end{pmatrix},$$

I_d is the $d\times d$ identity matrix and $0_{p\times q}$ is the $p\times q$ matrix with all entries 0. We define two block diagonal matrices $\mathcal{Q} := \text{blkdiag}(\overbrace{Q,\cdots,Q}^{T \text{ times}}, Q_f)$ and $\mathcal{R} := \text{blkdiag}(\overbrace{R,\cdots,R}^{T \text{ times}})$ derived from the given matrices Q, Q_f and R. The compact notation above allows us to state the following optimal control problem underlying the receding horizon control technique:

(5) $$\begin{aligned} &\underset{\pi_{t:T}}{\text{minimize}} && \mathbb{E}_{x_t}\left[\langle x_{t:T+1}, \mathcal{Q}x_{t:T+1}\rangle + \langle u^a_{t:T}, \mathcal{R}u^a_{t:T}\rangle\right] \\ &\text{subject to} && \begin{cases} \text{constraint (4)}, \\ u_t \in \mathbb{U} \text{ for all } t, \\ \pi_{t:N} \in \text{ a class of policies.}\end{cases}\end{aligned}$$

We impose the following blanket:

Assumption 1. The communication channel between sensors and the controller is noiseless.

2.2 Control policy class

We employ T-history-dependent policies of the form

(6) $$u_{t+\ell} = \eta_{t+\ell} + \sum_{i=0}^{\ell-1} \lambda_{\ell,t+i}\nu_{t+i},$$

for $\ell = 0,1,\ldots,T-1$.

The control vector $u_{t:T}$ admits the following form under the unreliable control channel:

(7) $$u_{t:T} = \begin{pmatrix}\eta_t \\ \eta_{t+1} \\ \vdots \\ \eta_{t+N-1}\end{pmatrix} + \mathbf{\Lambda}_t \begin{pmatrix}\nu_t \\ \nu_{t+1} \\ \vdots \\ \nu_{t+N-2}\end{pmatrix} =: \boldsymbol{\eta}_t + \mathbf{\Lambda}_t \nu_{t:T-1},$$

where $\boldsymbol{\eta}_t \in \mathbb{R}^{mT}$ and $\mathbf{\Lambda}_t \in \mathbb{R}^{mT\times(T-1)}$ is a strictly lower block triangular matrix

(8) $$\mathbf{\Lambda}_t = \begin{pmatrix} 0 & 0 & \cdots & 0 & 0 \\ \lambda_{1,t} & 0 & \cdots & 0 & 0 \\ \lambda_{2,t} & \lambda_{2,t+1} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \lambda_{T-1,t} & \lambda_{T-1,t+1} & \cdots & \lambda_{T-1,t+T-3} & \lambda_{T-1,t+T-2}\end{pmatrix},$$

with each $\lambda_{k,\ell} \in \mathbb{R}^m$.