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Stabilizing Model Predictive Control without Terminal Constraints for Switched Nonlinear Systems

Sven Knüfer * **Matthias A. Müller** * **Frank Allgöwer** *

* *Institute for Systems Theory and Automatic Control, University of Stuttgart, 70550 Stuttgart, Germany (e-mail: {sven.knuefer, matthias.mueller, frank.allgower} @ist.uni-stuttgart.de)*

Abstract: This work proposes a Model Predictive Control (MPC) approach without terminal constraints for switched nonlinear systems subject to time-dependent and a priori unknown switching signals. Under a controllability assumption, it is shown that the controlled system is asymptotically stable if the switching signal fulfills a certain average dwell time condition. The results are applied to a continuous stirred-tank reactor with two different modes of operation.

Keywords: Model Predictive Control, switched systems, nonlinear control

1. INTRODUCTION

Model Predictive Control (MPC) is a control technique, in which an open-loop optimal control problem is repeatedly solved. The main advantages of MPC include the possibility to explicitly consider input and state constraints and to apply it to general nonlinear systems. For computational reasons, a finite prediction horizon is used in the optimal control problem, which requires additional arguments to guarantee stability. A common procedure is to modify the optimal control problem by introducing a terminal cost and terminal region constraints, see, e.g., Chen and Allgöwer (1998) or Mayne et al. (2000). On the contrary, in MPC without terminal constraints, the finite horizon optimal control problem remains unchanged and stability is, e.g., achieved by verifying certain controllability assumptions, see Grüne (2009) or Reble and Allgöwer (2012).

This work considers MPC without terminal constraints for the class of switched nonlinear systems. In this class of systems, the dynamics are determined by several subsystems and by a switching signal that indicates the active subsystem. Switched systems are of relevance in many applications, such as for switched control strategies, in reductions of nonlinear models to piecewise affine ones, or under instantly changing system characteristics or operating conditions, see, e.g., (Rantzer and Johansson, 2000; Morse, 1995; Liberzon, 2003). For general time-dependent, a priori unknown switching signals considered in this work, it is well known that, even if a switched system consists of stable subsystems only, it may be destabilized by certain switching signals (Liberzon, 2003). Hence, to guarantee stability, it may be necessary to restrict the set of allowed switching signals. Typical restrictions are formulated in terms of dwell time or average dwell time conditions, see Morse (1996) or Hespanha and Morse (1999). For switched nonlinear systems, these conditions are derived from multiple Lyapunov functions fulfilling a compatibility assumption (Liberzon, 2003).

In this work, we make use of the average dwell time framework to design a stabilizing MPC scheme for switched nonlinear systems. A stabilizing MPC scheme for this class of systems has also been developed in Mhaskar et al. (2005), where the switching signal is, however, required to be known a priori. In Müller et al. (2012), the authors discussed a setup that is closely related to this work by presenting an average dwell time condition for MPC *with* terminal constraints. In order to verify that the switched system is asymptotically stabilized by the MPC scheme, additional assumptions particularly for the switched setup are needed in Müller et al. (2012). As presented in this paper, the advantage of MPC *without* terminal constraints is that similar stability results for switching signals subject to a certain average dwell time condition are obtained nearly without additional assumptions. Precisely, this work shows that the usual assumptions to prove that a single subsystem is stable under MPC without terminal constraints only needs minor adjustments to get asymptotic stability in the switched case.

This work is structured as follows. In Section 2, the setup and our notation are presented. Section 3 introduces and analyzes a switched MPC algorithm. In Section 3.1, the main stability result for this algorithm is formulated in terms of compatibility and decrease conditions for the optimal value functions of the subsystems. In Section 3.3, we show how these conditions can be satisfied, using, among others, MPC stability results from the non-switched case, which are reviewed in Section 3.2. In Section 4, our results are applied to a continuous stirred-tank reactor. Finally, Section 5 gives a brief conclusion.

2. PRELIMINARIES AND SETUP

In this work, we consider a family of subsystems

$$\dot{x}(t) = f_p(x(t), u(t)), \quad t > 0,$$
$$x(0) = x_0,$$

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where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{U}$, with $\mathbb{U} \subseteq \mathbb{R}^m$ compact, are the state and the input signal at time t, respectively. Moreover, $p \in P$ is the subsystem index from a finite index set P. For each subsystem, we assume that the vector field f_p is locally Lipschitz and that the origin is an equilibrium point of the undriven system, i.e. $f_p(0,0) = 0$. In the following, we consider input signals $u \in C_{\mathrm{pw}}([0,\infty),\mathbb{U})$, where $C_{\mathrm{pw}}([0,\infty),\mathbb{U})$ denotes the set of all piecewise continuous functions which map from $[0,\infty)$ to $\mathbb{U}$.

In the setup considered here, an a priori unknown switching signal $\sigma : [0,\infty) \to P$ determines which subsystem $p \in P$ is currently active. The switching signal is assumed to be piecewise constant and right-continuous such that the switches occur at discrete points τ_k, with $\tau_0 = 0$ and $\tau_k < \tau_{k+1}$. Hence, the switched system is given by

$$\begin{aligned} \dot{x}(t) &= f_{\sigma(t)}(x(t), u(t)), \quad t > 0, \\ x(0) &= x_0, \end{aligned} \tag{1}$$

i.e. a solution $x(\cdot)$ follows the vector field $f_{\sigma(\tau_i)}$ on each interval $[\tau_i, \tau_{i+1})$ and it is continuous in any switch τ_i. In the following, we consider the class of switching signals exhibiting a certain average dwell time, as introduced by Hespanha and Morse (1999).

Definition 1. A switching signal σ has *average dwell time* $\tau_{\mathrm{a}} > 0$ if there exists a number $N_0 \geq 0$ such that

$$N_\sigma(t_1, t_2) \leq N_0 + \frac{t_1 - t_2}{\tau_{\mathrm{a}}}$$

holds for all $t_1 \geq t_2 \geq 0$, where $N_\sigma(t_1,t_2)$ denotes the number of switches in the interval $(t_2, t_1]$.

The goal is to control the switched system (1) with an MPC algorithm that does not make use of stabilizing terminal constraints. The following finite horizon open-loop optimal control problem is used for such an MPC scheme in the non-switched context. Hence, it is formulated for a single subsystem p.

Problem 2. For the initial state x_0 and the subsystem index p, solve the optimization problem

$$\min_{\bar{u} \in C_{\mathrm{pw}}([0,T],\mathbb{U})} J_{T,p}(x_0, \bar{u})$$

with

$$J_{T,p}(x_0, \bar{u}) = \int_0^T L_p(\bar{x}(t), \bar{u}(t)) \mathrm{d}t,$$

subject to

$$\begin{aligned} \dot{\bar{x}}(t) &= f_p(\bar{x}(t), \bar{u}(t)), \quad t \in [0,T], \\ \bar{x}(0) &= x_0. \end{aligned}$$

In Problem 2, the notation $\bar{u}(\cdot)$ denotes the predicted input sequence and $\bar{x}(\cdot)$ is the corresponding predicted state trajectory starting at the initial condition x_0. The stage cost function $L_p : \mathbb{R}^n \times \mathbb{U} \to [0,\infty)$ is chosen to be continuous and to satisfy $L_p(0,0) = 0$. Since Problem 2 does not include terminal constraints, any piecewise continuous input signal $\bar{u}$ defined on $[0,T]$ is feasible if it satisfies the input constraints $\bar{u}(t) \in \mathbb{U}$ and if it ensures that the predicted state trajectory $\bar{x}$ exists on the whole prediction interval $[0,T]$. Consequently the feasible set of Problem 2 can be rather large compared to MPC schemes *with* terminal constraints.

In the following, it is assumed that a minimizer $u^*_{T,p}(\cdot;x_0)$ with $J_{T,p}(x_0, u^*_{T,p}(\cdot;x_0)) = \min_{\bar{u} \in C_{\mathrm{pw}}([0,T],\mathbb{U})} J_{T,p}(x_0,\bar{u})$ exists for all initial states $x_0 \in \mathbb{R}^n$ and all subsystems $p \in P$. This allows to define the optimal value function $J^*_{T,p} : \mathbb{R}^n \mapsto [0,\infty)$ mapping an initial condition to its respective minimal value $J_{T,p}(x_0, u^*_{T,p}(\cdot;x_0))$. Moreover, the corresponding state trajectory is denoted by $x^*_{T,p}(\cdot;x_0)$.

Remark 3. In the presented setup, no state constraints are considered. Hence, initial and recursive feasibility follow immediately. On the other hand, in case that state constraints of the form $x \in \mathbb{X} \subset \mathbb{R}^n$ should be considered, modifications might be needed to guarantee recursive feasibility. Boccia et al. (2014) and Chapter 8 of Grüne and Pannek (2011) give detailed discussions about this aspect in the context of MPC without terminal constraints.

3. MODEL PREDICTIVE CONTROL FOR SWITCHED SYSTEMS

Problem 2 considers a certain subsystem with index p as the vector field f_p gives the dynamics and as the weighting is done by the stage cost function L_p, which is chosen appropriately for subsystem p. Nevertheless, the minimizing input signals of Problem 2 are also useful in the switched case. Since this work considers a priori unknown switching signals, a prediction is to be based on the currently active subsystem. To this end, it is assumed that all switches are detected instantly. This results in the following switched MPC algorithm, see Müller et al. (2012).

Algorithm 1. Initialize the algorithm with $i = k = 0$ and $t_0 = 0$. For each t_i execute the following steps.

(1) Measure the state $x(t_i)$ and determine $p_i := \sigma(t_i)$.
(2) Solve Problem 2 for $x_0 := x(t_i)$ and $p := p_i$.
(3) Set $u_{\mathrm{MPC}}(t) := u^*_{T,p_i}(t - t_i; x(t_i))$ for $t \in [t_i, t_{i+1})$, where $t_{i+1} := \min\{t_i + \delta, \tau_{k+1}\}$ defines the next sampling instant, i.e. apply the first part of the optimal control input $u^*_{T,p_i}(\cdot; x(t_i))$.
(4) Let $i := i+1$. If $t_i = \tau_{k+1}$, let $k := k+1$.

Since the future behavior of the switching signal is unknown, the sampling instants t_i are defined online. At these sampling instants, Problem 2 is solved anew in Step 3 either because the nominal sampling time δ has passed or because a switch occurs. Although different sampling times have to be considered in view of unknown switching signals, δ always denotes the fix nominal sampling time in the following. As indicated by Step 3, Algorithm 1 defines the input signal $u_{\mathrm{MPC}} \in C_{\mathrm{pw}}([0,\infty),\mathbb{U})$ and consequently the resulting trajectory is denoted by x_{MPC}.

3.1 Stability

For Algorithm 1, the following main result about asymptotic stability can be formulated.

Theorem 4. Suppose there exist $\lambda \in (0,\infty)$ and $\mu, C \in [1,\infty)$ such that for all $p,q \in P$, all $x_0 \in \mathbb{R}^n$, and all $d \in [0,\delta]$ the estimates

$$J^*_{T,p}(x^*_{T,p}(\delta;x_0)) \leq e^{-\lambda\delta} J^*_{T,p}(x_0), \tag{2}$$

$$J^*_{T,p}(x^*_{T,p}(d;x_0)) \leq C J^*_{T,p}(x_0), \tag{3}$$

and

$$J^*_{T,p}(x_0) \leq \mu J^*_{T,q}(x_0) \tag{4}$$