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IFAC-PapersOnLine 49-18 (2016) 104–109

Velocity Control of a Counter-Flow Heat Exchanger

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Abstract: In this paper we consider a nonlinear tracking regulation problem for a counter-flow heat exchanger using velocity control. The methodology is based on a variation of our earlier work on design of feedback laws for regulation of distributed parameter systems based on the geometric theory of output regulation. In previous work we developed a technique, referred to as the β-iteration method, that has been successfully applied to regulate a wide variety of linear and nonlinear control system applications. We demonstrate that a modification of the β-iteration method can be successfully applied to a problem of thermal regulation for a counter-flow heat exchanger using velocity control. In particular, the thermal regulation of the cold (or hot) fluid is achieved by changing the speed of the hot (or cold) fluid. In this example, the β-iteration method produces a nonlinear control input operator that strongly depends on the state of the system. We provide two numerical examples where the goal is to track a time dependent reference signal. These examples illustrate the rapid convergence of the method and we observe that two iterations are often sufficient to achieve very accurate practical regulation.

Keywords: Distributed Control, Counter-Flow Heat Exchanger, Regulation, Tracking, Velocity Control

1. INTRODUCTION

The problem of controlling a hyperbolic thermal flow system has been the topic of many research articles. In this paper we focus on a nonlinear control problem where the control input is the flow velocity. The papers [Maidi, A., et al. (2009), Gundepudi, P. K. and Friedly, J. C. (1998) and Shang, H., et al. (2005)] contain results on similar problems and provide a rather complete set of references on this topic. In particular, these papers focus on employing a velocity control mechanism for output tracking regulation. In this paper we also consider a tracking problem with flow velocity as the control and apply a modification of the geometric control design methodology as described in [Isidori, A. (1999)] and [Aulisa, E., Gilliam, D. S., (2015)]. After presenting a mathematical description of the control system and formulating the regulation problem in Section 2, we consider the set-point problem in order to draw attention to important limitations in using velocity control. In particular, the use of velocity control will involve certain physical constraints that limit the reference signals that can be tracked. We see that thermal regulation of the cold (or hot) fluid is obtained by changing the speed of the hot (or cold) fluid. From this observation it is clear that the thermal reference signal must have values that lie between the cold and hot fluids. But it is actually more complicated than that. Indeed, we observe that the resulting control input operator is non-linear and strongly depends on the state of the system.

In order to gain some insight into the possible tracking constraints, we first analyze the set-point problem for a constant reference signal $y_{r_0} \in \mathbb{R}$. To do this, in Section 3, we perform a dimensional analysis of the equations which will be used to study the set-point control problem. This analysis greatly simplifies the system and allows us to focus on the important parameters required to solve the control problem and provides a set of non-linear algebraic equations whose solution gives the desired control law. This allows us to give a very precise range of allowable values for the reference signal y_{r_0} in terms of the various physical parameters of the problem.

In Section 4, we investigate the β-iteration algorithm for tracking more general time dependent reference signals. The necessity of introducing the β-iteration is that applying the geometric control approach leads to a singular differential algebraic equation (DAE). Nevertheless, based on our previous work in [Aulisa, E., Gilliam, D. S., (2015)], the β-iteration algorithm provides a regularization scheme that can be used to obtain time dependent control laws which provide an approximate but accurate solution of the regulation problem.

In Section 5 we demonstrate how our modification of the β-iteration method can be successfully applied to a problem of thermal regulation for a counter-flow heat exchanger using velocity control. We present two specific numerical cases to illustrate the method and applicability of the method. In Example 1, we consider the problem of tracking

10.1016/j.ifacol.2016.10.147

of a reference signal consisting of a periodic perturbation of a constant set-point. The second Example 2, involves the tracking of a piecewise constant reference signal.

We note that a proof of convergence of the β-iteration algorithm for linear distributed parameter control systems was recently obtained, [Pathiranage, T. W., et al. (2015)], and we are currently working on a similar analysis for the non-linear case.

2. HEAT EXCHANGER MODEL AND NOTATION

We consider a one dimensional counter-flow heat exchanger of length L, with position variable x varying from 0 to L. The temperature of the fluids in the inner pipe (bottom) and outer pipe (top) at the point x and time t are denoted by $T_1(t,x)$ and $T_2(t,x)$, respectively. The velocities of the fluids are denoted by v_1 and v_2, and K_1 and K_2 are positive thermal constants. By convention we fix v_1 to be positive if the fluid in the inner pipe flows from the left to the right, and v_2 to be positive if the fluid in the outer pipe flows from the right to the left. We assume there is a hot (or cold) fluid with temperature $T_1(t,x)$ flowing from the left to the right with velocity v_1 in the inner pipe, and a cold (or hot) fluid $T_2(t,x)$ flowing from the right to the left with velocity v_2 in the outer pipe, as depicted in Figure 1. Further, we assume that the fluid enters the inner pipe on the left with given temperature d_1, modeled as a Dirichlet boundary condition, $T_1(t,0) = d_1 > 0$, while the fluid enters into the outer pipe on the right with given temperature d_2, modeled again as a Dirichlet boundary condition $T_2(t,L) = d_2 > 0$. Without loss of generality, we assume $d_1 > d_2$, and consequently fluid in the inner pipe is the hot fluid, while and fluid in the outer pipe is the cold one.

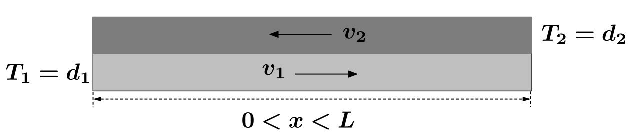

Fig. 1. Counter-Flow Heat Exchanger.

Thus, we consider the following simple hyperbolic model

$$\frac{\partial T_1}{\partial t} = -v_1 \frac{\partial T_1}{\partial x} + K_1(T_2 - T_1)\,, \tag{1}$$

$$\frac{\partial T_2}{\partial t} = v_2 \frac{\partial T_2}{\partial x} + K_2(T_1 - T_2)\,, \tag{2}$$

with initial conditions

$$T_1(0,x) = h(x), \tag{3}$$

$$T_2(0,x) = c(x), \tag{4}$$

and boundary conditions

$$T_1(t,0) = d_1, \quad T_2(t,L) = d_2\,. \tag{5}$$

The Measured Output: We consider as measured output the temperature T_1 at a point $0 < x_1 \le L$

$$y(t) = T_1(t,x_1). \tag{6}$$

Note that this is an idealize (unbounded) output.

The Control Input: We consider a control input entering through a velocity control of the fluid 2 as

$$v_2 = v_1 + u. \tag{7}$$

Replacing $v_2 = v_1 + u$ in the second equation the system (1)-(4) becomes

$$\frac{\partial T_1}{\partial t} = -v_1 \frac{\partial T_1}{\partial x} + K_1(T_2 - T_1), \tag{8}$$

$$\frac{\partial T_2}{\partial t} = (v_1 + u) \frac{\partial T_2}{\partial x} + K_2(T_1 - T_2), \tag{9}$$

together with initial conditions

$$T_1(0,x) = h(x), \tag{10}$$

$$T_2(0,x) = c(x), \tag{11}$$

and boundary conditions

$$T_1(t,0) = d_1, \quad T_2(t,L) = d_2. \tag{12}$$

Notice that the disturbances d_1 and d_2 enter the system through unbounded input operators in the Hilbert state space $L^2((0,L),\mathbb{R}^2)$.

The Control Problem: Our objective is to force the measured output $y(t)$ to track a time dependent reference temperature $y_r(t) \in \mathbb{R}$. Thus, we want to construct a velocity control $u(t)$ such that the error

$$e(t) = y(t) - y_r(t) \xrightarrow{t\to\infty} 0.$$

We reformulate the problem (8)-(12) in operator form. Define

$$T = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}, \quad T_0 = \begin{bmatrix} h \\ c \end{bmatrix},$$

$$B_{\mathrm{d},1} d_1 = \begin{bmatrix} -\delta_0 \\ 0 \end{bmatrix} d_1, \quad B_{\mathrm{d},2} d_2 = \begin{bmatrix} 0 \\ \delta_L \end{bmatrix} d_2,$$

where δ_a denotes the Dirac delta distribution located at $x = a$. The control input operator, which depends strongly on the state of the system, is given by

$$B_{\mathrm{in}} T = \begin{bmatrix} 0 \\ \dfrac{d}{dx} T_2 \end{bmatrix}.$$

The state operators are defined by

$$A_v = \begin{bmatrix} -v_1 \dfrac{d}{dx} & 0 \\ 0 & \dfrac{d}{dx} v_1 \end{bmatrix},$$

$$A_K = \begin{bmatrix} -K_1 & K_1 \\ K_2 & -K_2 \end{bmatrix}, \quad A = A_v + A_K,$$

where $\mathcal{D}(A)$ consists of all pairs

$$\left\{ \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} : \varphi_1, \varphi_2 \in H^1(0,L),\ \varphi_1(0) = \varphi_2(L) = 0 \right\}.$$

The output is given in terms of the unbounded observation operator

$$y(t) = CT = T_1(t,x_1), \quad 0 < x_1 \le L.$$

Using this notation, the control system can be rewritten (at least formally) in the typical operator form

$$\frac{dT}{dt}(t,x) = AT(t,x) + B_{\mathrm{d}} + B_{\mathrm{in}}(t,x)u(t), \tag{13}$$

$$T(0,t) = T_0(x), \tag{14}$$

$$y(t) = CT(t), \tag{15}$$

where we have used the notation

$$B_{\mathrm{d}} = B_{\mathrm{d},1} d_1 + B_{\mathrm{d},2} d_2.$$

In Section 4 below, we use this formulation and apply a generalization of the β-iteration algorithm to solve the