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Estimation of a gaseous release into the atmosphere using a formation of UAVs

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Abstract: The real-time estimation of a gaseous plume that results from gas released into the atmosphere by a moving aerial source is presented. The approach is utilizing a group of UAVs equipped with gas concentration sensors onboard and the concentration measurements are used in an estimation scheme in the form of a Luenberger observer. The estimator is modeled as the advection-diffusion equation, which is solved numerically in real-time using a finite volume method on a dynamically adapted computational grid. The UAVs maintain a rigid flying formation, with the control signals for the leader being provided by the Lyapunov-redesign method, which couples the UAV dynamics to the performance of the estimator. The follower UAVs provide plant information to the estimator and also compute the spatial gradients necessary for the implementation of the guidance scheme. Numerical examples illustrate the performance of the proposed approach.

Keywords: Distributed parameter systems; Computational methods; UAVs; Sensor networks.

1. INTRODUCTION

The accidental gas release from a moving source can have adverse environmental implications. Similarly, deliberate gas release due to adversarial action can severely compromise military defenses. The first step in managing such a threat is to estimate the resulting concentration field and to localize the source using mobile sensors.

The problem of gaseous source localization using mobile robots was addressed in earlier works by Neumann et al. (2013); Rao (2007); Lilienthal et al. (2006), in which the sensor motion was dictated by a certain heuristic approach, or by Tzanos and Zefran (2007); Zhao and Nehorai (2004); Sivergina et al. (2003); Porat and Nehorai (1996) with a gradient-based scheme for the sensor guidance. The estimation of the concentration field was implemented in Šmídl and Hofman (2013); Kuroki et al. (2010), assuming known source location. More recently Zhang et al. (2011) considered adaptive sampling methods with an EnKF in which the vehicle control was found via the gradient of a quadratic measure using adjoint analysis.

In this work we continue developing an approach for the estimation of the concentration field in real time using UAVs, provided that the source is moving along an unknown trajectory. In our most recent work Egorova et al. (in print, 2016) this approach was implemented numerically with a *single* UAV equipped with the concentration sensor, assuming that the sensor provides information of *both* concentration *and* concentration gradients at the sensor location. The gaseous plume was modeled by the 3D advection-diffusion equation, which was subsequently solved numerically using the finite-volume method with a total variation diminishing (TVD) scheme. The UAV dynamics were described using the point-mass model of a fixed-wing aircraft. The desired inertial velocities for the UAV were chosen in terms of the concentration estimation error and the error gradients through the appropriate choice of a Lyapunov functional. The computational grid for the estimator was adapted dynamically in order to achieve greater computational efficiency. In this work, this approach is implemented using a rigid formation of UAVs whose utility is twofold. First, the concentration gradients appearing in the desired inertial velocities of the leader UAV, and thus in its guidance, are computed via the use of the point concentration measurements from the "follower"-UAVs that form a cross-shape formation around the "leader" UAV as shown in Figure 3. Secondly, the estimator model is modified to incorporate the additional concentration measurements provided by the follower UAVs.

The remainder of this paper is organized as follows: Section 2 provides the mathematical model for the gaseous release process (plant) and the estimator modified to account for the additional plant information from the follower UAVs, as well as the UAV dynamical model and guidance scheme. Section 3 summarizes the numerical implementation procedure of the proposed approach. The simulation results are given in Section 4, followed by conclusions in Section 5.

2. MATHEMATICAL MODEL

2.1 Process Model

Consider a source of gas release moving inside a 3D spatial domain $\Omega = [0, L_X] \times [0, L_Y] \times [0, L_Z] \in \mathbb{R}^3$. The gas source is characterized by a release rate $u(t)$ and a spatial distribution. The latter is given by the 3D Dirac measure concentrated at the source's centroid

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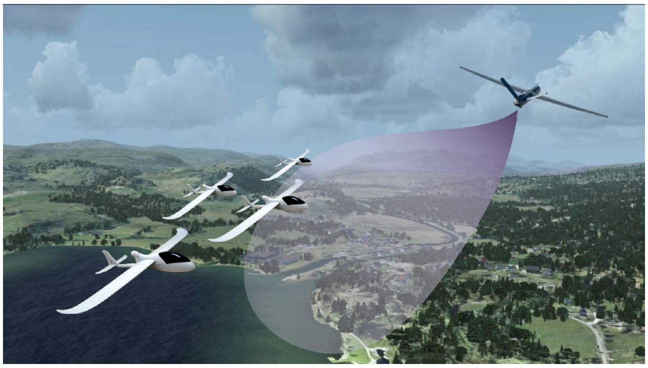

Fig. 1. Overview of the considered problem with an aerial source and a group of UAVs.

$\Theta_c(t) = (X_c(t), Y_c(t), Z_c(t))$. Therefore, the gas source is modeled mathematically as follows

$$\mathcal{S}(t, \Theta_c(t)) = \delta(X - X_c(t))\delta(Y - Y_c(t))\delta(Z - Z_c(t))u(t) \quad (1)$$

The plume is represented by the 3D advection-diffusion equation for the mean concentration $\langle c\rangle(t, X, Y, Z)$ Arya (1999),

$$\begin{aligned}\frac{\partial(\langle c\rangle(t,X,Y,Z)U)}{\partial t} &= -\frac{\partial(\langle c\rangle(t,X,Y,Z)U)}{\partial X} \\ &-\frac{\partial(\langle c\rangle(t,X,Y,Z)V)}{\partial Y} - \frac{\partial(\langle c\rangle(t,X,Y,Z)W)}{\partial Z} \\ &+\frac{\partial}{\partial X}\left(K_{XX}\frac{\partial\langle c\rangle(t,X,Y,Z)}{\partial X}\right) \\ &+\frac{\partial}{\partial Y}\left(K_{YY}\frac{\partial\langle c\rangle(t,X,Y,Z)}{\partial Y}\right) \\ &+\frac{\partial}{\partial Z}\left(K_{ZZ}\frac{\partial\langle c\rangle(t,X,Y,Z)}{\partial Z}\right) + \mathcal{S}(t, \Theta_c(t)),\end{aligned}$$

written in the abstract form as

$$\dot{\mathbf{x}}(t) = \mathcal{A}\mathbf{x}(t) + \mathcal{S}(t, \Theta_c), \quad (2)$$

where $\mathbf{x}(t) = \langle c\rangle(t, \cdot, \cdot, \cdot)$ is the concentration state, and $\mathcal{A}$ is the advection-diffusion operator:

$$\begin{aligned}\mathcal{A}\varphi = &-\frac{\partial(\varphi U)}{\partial X} - \frac{\partial(\varphi V)}{\partial Y} - \frac{\partial(\varphi W)}{\partial Z} + \frac{\partial}{\partial X}\left(K_{XX}\frac{\partial\varphi}{\partial X}\right) \\ &+\frac{\partial}{\partial Y}\left(K_{YY}\frac{\partial\varphi}{\partial Y}\right) + \frac{\partial}{\partial Z}\left(K_{ZZ}\frac{\partial\varphi}{\partial Z}\right).\end{aligned} \quad (3)$$

for all test functions $\varphi \in L_2(\Omega)$. Here $U, V,$ and W are the mean wind speeds and K_{XX}, K_{YY}, and K_{ZZ} are the nonzero components of the eddy diffusivity tensor in the X, Y and Z directions, respectively. The wind speed and eddy diffusivity are given functions of the coordinates $X, Y,$ and Z. The advection-diffusion PDE is supplemented with Neumann boundary conditions $\frac{\partial\langle c\rangle(t,X,Y,Z)}{\partial n}|_{\partial\Omega} = 0$ and an initial condition $\langle c\rangle(0, X, Y, Z) = 0$. The boundary conditions are incorporated in the definition of the domain of the spatial operator (3) and the initial condition for the evolution equation (2) is $\mathbf{x}(0) = 0$.

2.2 Estimator Model

The gas concentration sensors are attached to the center of mass of each of the UAVs. The state estimation framework is based on in situ Lagrangian sensing technique, according to which the sensors move freely in the fluid and gather measurements as they move through the environment Bennett (2006). By analogy with the source term, the sensor spatial distribution is described mathematically by the 3D Dirac delta function with centroid $\Theta_s(t) = (X_s(t), Y_s(t), Z_s(t)) \in \Omega$. The output operator associated with the sensor measurements may be written as follows

$$\begin{aligned}\mathcal{C}\varphi = &\int_0^{L_X}\int_0^{L_Y}\int_0^{L_Z} \varphi(X, Y, Z) \times \\ &\times \delta(X - X_s(t), Y - Y_s(t), Z - Z_s(t))\mathrm{d}X\mathrm{d}Y\mathrm{d}Z.\end{aligned} \quad (4)$$

for all test functions $\varphi \in L_2(\Omega)$. The estimator is essentially a copy of the advection-diffusion equation (2). It takes the form of a Luenberger observer supplemented with an output injection term, which is specified by the difference between a "true" (measured) concentration and a state estimate at the current sensor locations, multiplied by the filter gain matrix Γ and the dual $\mathcal{C}^*$ of the output operator $\mathcal{C}$ evaluated at the current sensor location

$$\dot{\hat{\mathbf{x}}} = \mathcal{A}\hat{\mathbf{x}} + \mathcal{C}^*\Gamma\mathcal{C}(\mathbf{x} - \hat{\mathbf{x}}), \quad (5)$$

where $\hat{\mathbf{x}}(t) = \langle\hat{c}\rangle(t, \cdot, \cdot, \cdot)$ denotes the estimated concentration state. Unlike the earlier effort Egorova et al. (in print, 2016), the filter gain Γ is now a positive definite matrix

$$\Gamma = \begin{bmatrix} \Gamma_{11} & 0 & \cdots \\ \vdots & \ddots & 0 \\ 0 & \cdots & \Gamma_{mm} \end{bmatrix} \quad (6)$$

where m is the number of sensors. Here $m = 7$ which accounts for the leader UAV (numbered as 4) and the six follower UAVs in the 3D rigid formation (cross-shape) of Figure 3. Please note that a full gain matrix Γ would require information exchange between *all* UAVs whereas a diagonal gain matrix Γ significantly reduces the communication requirements at the possible expense of estimator performance.

2.3 UAV Dynamics

Figure 2 shows the free-body diagram for a UAV climbing at a flight path angle γ and a bank angle ϕ. The derivation of the UAV equations of motion are based on the point-mass model of a fixed-wing aircraft Beard and McLain (2012); Zhao and Tsiotras (2010); Menon et al. (2012). Choosing the coordinate axes X, Y and Z to be directed east, north, and towards the earth center respectively, the equations of motion are given by

$$\begin{aligned}\dot{X} &= V_g \cos\gamma\cos\chi \\ \dot{Y} &= V_g \cos\gamma\sin\chi \\ \dot{Z} &= -V_g \sin\gamma \\ \dot{V}_g &= \frac{1}{M}(T - D(C_L) - Mg\sin\gamma) \\ \dot{\gamma} &= \frac{1}{MV_g}(L(C_L)\cos\phi - Mg\cos\gamma) \\ \dot{\chi} &= \frac{L(C_L)\sin\phi\cos(\chi - \psi)}{MV_g\cos\gamma}\end{aligned} \quad (7)$$