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Event-triggered control via reset control systems framework

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Abstract: This paper deals with a systematic way to design the event-triggered rules to stabilize a class of linear reset control systems. The event-triggering condition depends only on local information, that is it only uses the measured signals. The approach proposed combines a hybrid framework to describe the sampled-data system with Lyapunov-based techniques. Dwell-time dependent constructive conditions expressed through linear matrix inequalities (LMI) are proposed to design the event-triggered rule ensuring the asymptotic stability of the closed-loop system. The effectiveness of the approach is evaluated through an example borrowed from the literature.

Keywords: Reset control systems, hybrid systems, event-triggered control, Lyapunov techniques

1. INTRODUCTION

In recent years, sampled-data control designs for linear or nonlinear plants have been studied through several works. Hence, robust stability analysis with respect to aperiodic sampling has been widely studied (see, for example, Chen and Francis (1995); Heemels et al. (2010); Nešić and Teel (2004) and references therein), where variations on the sampling intervals are seen as a disturbance to the periodic case. The objective is then to provide an analysis of such systems using the discrete-time approach (Heemels et al. (2010); Cloosterman et al. (2010)), the input delay approach (Fridman et al. (2004); Seuret (2012)), or the impulsive systems approach (Naghshtabrizi et al. (2008)). Furthermore, an alternative and interesting vision of sampled-data systems has been proposed in Årzén (1999); Åström and Bernhardsson (1999), suggesting to adapt the sampling sequence to certain events related to the state evolution (see, for example, Aström (2008); Heemels et al. (2013); Hespanha et al. (2007); Lunze and Lehmann (2010); Tabuada (2007); Zampieri (2008)). This is called "event-triggered sampling", which naturally mixes continuous and discrete-time dynamics. Thus, the event-triggered algorithm design can be first rewritten as the stability study of a hybrid dynamical system, which has been carried out in different contexts in Goebel et al. (2009, 2012); Prieur et al. (2007, 2010).

In the context of event-triggered control, two objectives can be pursued: 1) the controller is a priori designed and only the event-triggered rules have to be designed, or 2) the joint design of the control law and the event-triggering conditions has to be performed. The first case is called the emulation approach, whereas the second one corresponds to the co-design problem. A large part of the existing works is dedicated to the design of efficient event-triggering rules, that is the designs done by emulation: see, for example, Heemels et al. (2012), Wang and Lemmon (2008), Postoyan et al. (2011), Tallapragada and Chopra (2012), Abdelrahim et al. (2014b) and references therein. Moreover, most of the result on event-triggered control consider that the full state is available, which can be unrealistic from a practical point of view. Hence, it is interesting to address the design of event-triggered controllers by using only measured signals. Some works have addressed this challenge as, for example, in Sbarbaro et al. (2014) in which the dynamic controller is an observer-based one, Abdelrahim et al. (2014a), in which the co-design of the output feedback law and the event-triggering conditions is addressed by using the hybrid framework.

The results proposed in the current paper take place in the context of the emulation approach, when the predesigned controller is issued from a hybrid dynamic output feedback controller, with the aim of using only the available signals. The controller under consideration is a reset control system (see Fichera et al. (2012), Fichera et al. (2013)). Actually, the approach proposed combines a hybrid framework to describe the sampled-data system with Lyapunov-based techniques. Constructive conditions, in the sense that linear matrix inequality (LMI) conditions are associated to a convex optimization scheme, are proposed to design the event-triggered rule ensuring asymptotic stability of the closed-loop system. Furthermore, differently from Abdelrahim et al. (2014a), a condition involving the allowable maximal sampling period T can be deduced by solving

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a set of LMIs proposed using a similar approach as in Mazo et al. (2010). Let us also emphasize that differently from most of the results in the literature, a reset rule is considered in our approach. Through an illustrative example borrowed from the literature, we point out the interest of the reset control law to reduce the number of control updates.

The paper is organized as follows. In Section 2, the system under consideration together with the sampled-data architecture is defined. Describing the associated dynamical hybrid system, the problem we intend to solve is formally stated. Section 3 is dedicated to presenting the main conditions, allowing to design the event-triggering rules. The condition to design the associated dwell-time is also derived. Section 4 illustrates the results and compares them with some existing approach. Finally, in Section 5, some concluding remarks end the paper.

Notation. The sets $\mathbb{N}$, $\mathbb{R}^+$, $\mathbb{R}^n$, $\mathbb{R}^{n\times n}$ and $\mathbb{S}^n$ denote respectively the sets of positive integers, positive scalars, n-dimensional vectors, $n \times n$ matrices and symmetric matrices in $\mathbb{R}^{n\times n}$. For a matrix P in $\mathbb{S}^n$, the notation $P \geq 0$ ($P > 0$) means that P is symmetric positive (definite). The superscript 'T' stands for matrix transposition, and the notation $\mathrm{He}(P)$ stands for $P + P^T$. The Euclidean norm is denoted $|\cdot|$. Given a compact set $\mathcal{A}$, the notation $|x|_{\mathcal{A}} = \min\{|x-y|,\ y \in \mathcal{A}]\}$ indicates the distance of the vector x from the set $\mathcal{A}$. The symbols I and 0 represent the identity and the zero matrices of appropriate dimensions.

2. PROBLEM STATEMENT AND SAMPLED-DATA ARCHITECTURES

2.1 Reset control systems

Consider a continuous-time reset control system described by

$$\begin{cases} \dot{x}_p = A_p x_p + B_p u_p, \\ \dot{x}_c = A_c x_c + B_c y_p, \end{cases} \quad (x_p, x_c) \in \mathcal{C},$$
$$\begin{cases} x_p^+ = x_p, \\ x_c^+ = J_p y_p + J_c x_c, \end{cases} \quad (x_p, x_c) \in \mathcal{D}, \tag{1}$$
$$\begin{cases} u_p = C_c x_c + D_c C_p x_p, \\ y_p = C_p x_p, \end{cases}$$

where $x_p \in \mathbb{R}^{n_p}$, $x_c \in \mathbb{R}^{n_c}$, $u_p \in \mathbb{R}^{m_p}$ and $y_p \in \mathbb{R}^{p_p}$ stand respectively for the state variable of the plant, the state of the dynamic controller, the input and the output of the plant. The matrices $J_p, J_c, A_p, B_p, C_p, A_c, B_c, C_c$ and D_c are constant and given matrices of appropriate dimensions. $\mathcal{C}$ and $\mathcal{D}$ are the flow and jump sets, that are usually defined as

$$\mathcal{C} = \left\{(x_p, x_c) :\ \begin{bmatrix} y_p \\ x_c \end{bmatrix}^T \bar{M} \begin{bmatrix} y_p \\ x_c \end{bmatrix} \leq 0\right\}, \tag{2a}$$
$$\mathcal{D} = \left\{(x_p, x_c) :\ \begin{bmatrix} y_p \\ x_c \end{bmatrix}^T \bar{M} \begin{bmatrix} y_p \\ x_c \end{bmatrix} \geq 0\right\}, \tag{2b}$$

where matrix $\bar{M} \in \mathbb{R}^{(p_p+n_c)\times(p_p+n_c)}$ is a design parameter.

Such a system can appear when we connect, for instance, a linear continuous plant with a reset controller (see Fichera et al. (2012), Fichera et al. (2013)). Then, to study this kind of systems, the hybrid formalism of Goebel et al. (2009); Prieur et al. (2007, 2013) can be used.

In this paper we deal with the problem of event-triggered implementation of a stabilizing control law connected to a linear continuous plant. Then, we will show that this kind of problem can be performed by using the framework associated to system (1). Furthermore, to particularize system (1) to the sampled-data architecture, we will see that defining an augmented state composed of the state of the closed-loop system (i.e. x_p and x_c) and the variables due to the sampled part (i.e. the held value of the control input and a timer), the problem consists in designing the sets $\mathcal{C}$ and $\mathcal{D}$.

2.2 Hybrid representation of sampled-data systems

A sampled-data implementation of a control law u_p and of the plant output y_p corresponds to breaking the continuous-time closed loop given by $s(t) = \begin{bmatrix} u_p(t) \\ y_p(t) \end{bmatrix}$, for all $t \geq 0$, and converting this into a zero order hold $\dot{s} = 0$ combined with the update rule $s^+ = \begin{bmatrix} u_p \\ y_p \end{bmatrix}$, which should be performed at suitable times according to the specific sampled-data architecture.

Event-triggered sampling corresponds to performing the update rule s^+ whenever the augmented state (x_p, x_c, s) belongs to suitable sets that should be designed in such a way to guarantee asymptotic stability of the closed-loop sampled-data system. In this case, the sampled-data system may be represented similarly to system (1) by the following dynamics, augmented with a timer σ used to induce a desirable dwell-time between each pair of consecutive samplings:

$$\begin{cases} \dot{x}_p = A_p x_p + B_p s_p, \\ \dot{x}_c = A_c x_c + B_c s_c, \\ \dot{s}_p = 0, \\ \dot{s}_c = 0, \\ \dot{\sigma} = g_T(\sigma), \end{cases} \quad (x_p, x_c, s_p, s_c, \sigma) \in \mathcal{C},$$
$$\begin{cases} x_p^+ = x_p, \\ x_c^+ = J_p C_p x_p + J_c x_c, \\ s_p^+ = C_c x_c + D_c C_p x_p, \\ s_c^+ = C_p x_p, \\ \sigma^+ = 0, \end{cases} \quad (x_p, x_c, s_p, s_c, \sigma) \in \mathcal{D}. \tag{3}$$

Timer $\sigma \in [0, 2T]$ flows by keeping track of the elapsed time since the last sample (where it was reset to zero) according to the following set-valued dynamics:

$$g_T(\sigma) = \begin{cases} 1 & \sigma \leq 2T \\ [0,1] & \sigma = 2T, \end{cases}$$

whose rationale is that whenever $\sigma < 2T$, its value exactly represents the elapsed time since the last sample, moreover $\sigma \in [T, 2T]$ implies that at least T seconds have elapsed since the last sample. [1] In (3), the so-called flow and jump sets $\mathcal{C}$ and $\mathcal{D}$ must be suitably selected to induce a desirable behavior on the sampled-data system and are the available degrees of freedom in the design of the event-triggered algorithm.

[1] Note that the use of a set-valued map for the right hand side g_T of the flow equation for σ enables us to confine the timer σ to a compact set $[0, 2T]$, while at the same time using dynamics whose right hand sides are outer semicountinuous set-valued mappings, thereby satisfying the regularity conditions in (Goebel et al., 2012, As. 6.5) and enjoying the desirable robustness properties of stability of compact attractors established in (Goebel et al., 2012, Ch. 7).