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Adjoint-based state and distributed parameter estimation in a switched hyperbolic overland flow model*

Van Tri NGUYEN* Didier GEORGES* Gildas BESANÇON*

* *Univ. Grenoble Alpes, GIPSA-Lab, F-38000 Grenoble, France.*
E-mail: {van-tri.nguyen, didier.georges, gildas.besancon}@gipsa-lab.grenoble-inp.fr

Abstract: This paper considers the issue of state and parameter estimation in an overland flow model, including unknown infiltration coefficients. The overland flow dynamics are described by the continuity Saint-Venant equation, and the infiltration by the Green-Ampt model. Due to the related so-called 'ponding time', the overall model results in a switched partial differential equation, coupled with an ordinary differential one. In this model, the initial state, the friction coefficient and the infiltration parameters are assumed to be unknown, with only a discrete number of measurements being available. For the estimation, the model is modified by adding an activation function, and then used in the minimization of a cost function defined as the difference between available measurements and the corresponding simulated ones. The variational analysis is applied on the augmented Lagrangian objective functional in order to get the weak form of gradients of this function with respect to the variables to be estimated. Based on these gradients a quasi Newton method is used to solve the optimization problem. An illustration example is finally provided to validate the proposed approach.

Keywords: Switched Systems; Switched overland flow model; Green - Ampt model; State estimation; Distributed parameter estimation; Switched hyperbolic systems; Adjoint method; Inverse problem; Saint-Venant model;

1. INTRODUCTION

Overland flow or surface runoff is a very common watering process and plays an importance role in the water cycle. Being caused by the rainfall, snowfall or glaciers melting, the overland flow occurs when the soil is saturated to full absorption capacity. It is mainly driven by two forces: gravity and capillary action. The remaining water, after infiltration, will flow on the soil surface, and down to some channels, as shown in Figure 1. In this contribution, we only consider the overland flow generated by rainfall. The dynamics of the flow are described by using the continuity equation of Saint-Venant model, while the infiltration process is characterized by the common Green-Ampt physical model (see Heber Green and Ampt (1911)). The infiltration rate depends on a lot of factors like soil texture and structure, soil surface vegetation, initial watering state of soil, and rainfall rate as well. Basically, an infiltration process can be divided into two phases, before and after a so-called 'ponding time'. This time is a function of some physical parameters describing the mentioned state and characteristics of soil at infiltration moment. By considering that these parameters are unknown, as well as the initial state and friction coefficient of the continuity Saint-Venant equation, we hereby propose an estimation method based on adjoint analysis. The estimation problem of hydrologic parameters, including Manning friction coefficient or Green-Ampt parameters has been addressed by various works in both hydrologic and automatic control domains. Becker et al.(see Becker and Yeh (1972)) for instance used influence coefficient algorithm to estimate the friction in an unsteady open-channel flow. The minimization of least square error between the observed and simulated value has been used by Han Longxi to estimate the roughness coefficient and its application in a channel network (see Longxi (2008)). Y. Ding et al. proposed adjoint method for identifying the same parameter in shallow water flows

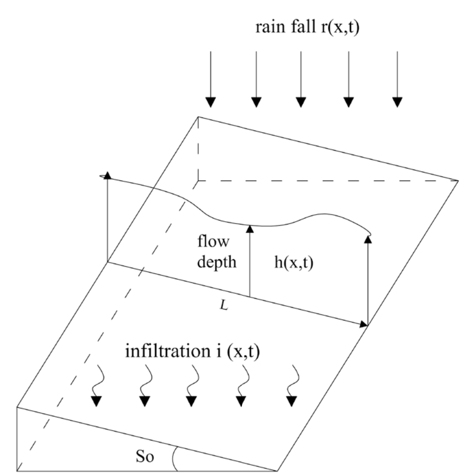

Fig. 1. Overview of an overland flow cause by rainfall.

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and channel network (see Ding et al. (2004)). We also investigated the adjoint method combined with Radial Basis Function Network framework to identify the optimal value of a distributed Manning coefficient (see Nguyen et al. (2015a)). A validation of adjoint method on the estimation for this variable has also been provided by using real measurements in Nguyen et al. (2015b). On the other hand, the identification of Green-Ampt parameters has been studied mainly by hydrologists in some researches such as: Brakensiek et al., Esen used fitting procedure to get the values from infiltrometer data, presented in (see Brakensiek and Onstad (1977)) and (see Esen (1989)); Kidwell et al. (see Kidwell et al. (1997)) estimated the effective hydraulic conductivity by using model minimization on Water Erosion Prediction Project, while Santos et al. used adjoint method; more recently, Castaings et al. analyzed the potential of variational approach for parameter estimation for distributed overland flow (see Castaings et al. (2009)); an investigation of the Green–Ampt parameters estimation from large data set of rainfall simulation data has been carried out in the work of van den Putte et al. (see Van den Putte et al. (2013)); in 2015, Li Chen et al. proposed an optimization method to minimize the root-mean- square-error-based objective function under rainfall conditions to get the values of Green-Ampt parameter (see Chen et al. (2015)). There are also some related works for Saint Venant equations: we have studied the adjoint method in this context in Nguyen et al. (2014); Rafiee et al. used the extended Kalman filter in Rafiee et al. (2011) and the sequential Monte Carlo methods for state estimation in large-scale open channel networks (see Rafiee et al. (2013)).

Following all those studies, we consider here an issue of simultaneous state and parameter estimation problem. More precisely, we propose to identify the Manning coefficient here assumed to be spatially distributed, the parameters of the Green-Ampt infiltration model, and the initial state of water flow depth. Thus the problem combines state estimation, with unknown constant coefficients, as well as some unknown distributed one. In addition, the ponding time induces a switch in the infiltration model. To our knowledge this overall problem has not been studied so far, and our purpose here is to propose solution based on an adjoint method in the spirit of our former studies of (Nguyen et al., 2015b) or (Nguyen et al., 2016). To that end, the model is first modified by including a continuous activation function (depending on some of the unknown variables), and an optimization procedure is then considered.

The rest of this work is organized as follows. In section 2, the switched overland flow based on Saint-Venant equation and Green-Ampt infiltration model is presented. In section 3, an adjoint-based estimation method is proposed. The numerical implementation is described in section 4. Finally, a conclusion is given in the last one.

2. SWITCHED OVERLAND FLOW MODEL

By using some suitable approximations (see Nguyen et al. (2014) for more details) , the dynamics of a one dimensional overland flow on a catchment can be described by the continuity equation of Saint-Venant model as follows:

$$\begin{cases} \dfrac{\partial h(x,t)}{\partial t} + \dfrac{\partial f(h(x,t),x)}{\partial x} = r(x,t) - i(x,t) \\ h(x,0) = h_0^i(x) \quad \text{and} \quad h(0,t) = h_0^b(t) \end{cases} \tag{1}$$

where x = position, (m); t = time, (s); $h(x,t)$ = water flow depth, (m); $f(h(x,t),x) = h^{5/3}S_0^{1/2}/n(x)$ = flow per width unit (m^2/s) with the Manning roughness coefficient $n(x)$ $s/m^{1/3}$; $r(x,t)$ = variable rainfall rate, (m/s); $i(x,t)$ = infiltration rate, (m/s); S_0 = bed slope, (m/m); $h_0^i(x)$ and $h_0^b(t)$ are respectively the initial condition and boundary condition. The rainfall rate $r(x,t)$ and infiltration rate $i(x,t)$ are supposed to be uniform all over the considered spatial domain while the Manning coefficient, an important empirical parameter (affected by a lot of soil and surface factors) is assumed to be a function of space variable x. This means that the rainfall and infiltration rates can be denoted as $r(t)$, $i(t)$ and the notation of the Manning coefficient is $n(x)$.

The infiltration rate i, in system equation (1), is characterized by the classical Green-Ampt model (see Heber Green and Ampt (1911)). This is the first physically based model to describe the infiltration of water into soil under the rainfall condition. Due to its simplicity and performance, this model is widely used in many domains such as soil physics, hydrology. The evolution of infiltration rate can be divided into two periods: before and after a special time, called 'ponding time' as in equation (2).

$$\begin{cases} i(t) = K_i\left(\dfrac{\Psi\eta(1-\theta)}{I(t)} + 1\right) & \text{if } \ t > t_p \\ i(t) = r & \text{if } \ t \le t_p \end{cases} \tag{2}$$

where K_i = effective hydraulic conductivity of the soil, (m/s); Ψ = soil suction at wetting front, (m); η = soil porosity, $(\%)$; θ = relative initial soil moisture, $(unitless)$; $I(t)$ = cumulative infiltration defined by the integral of infiltration rate $I(t) = \int_0^t i(\tau)d\tau$; it is the accumulated depth of water infiltrating during the time period t, (m); t_p = ponding time, (s). At the begin of the process, the infiltration rate i is equal to the rainfall rate, and the cumulative infiltration increases to the saturated point of soil. This moment is the ponding time t_p, which depends on rainfall rate and soil characteristics. It is calculated by equation 3.

$$t_p = \frac{\Psi K_i \eta(1-\theta)}{r(r-K_i)} \tag{3}$$

Because of the ponding time, the overland flow system can be considered as a switched partial differential equation (PDE) system coupled with an ordinary differential equation (ODE) as in equation (4), with switching time t_p:

$$\begin{cases} \dfrac{\partial h}{\partial t} + \dfrac{\partial f(h,x)}{\partial x} = 0 & \text{if } t \le t_p \\ \dfrac{\partial h}{\partial t} + \dfrac{\partial f(h,x)}{\partial x} = r - K_i\left(\dfrac{\Psi\eta(1-\theta)}{I(t)} + 1\right) & \text{if } t > t_p \\ \dfrac{dI(t)}{dt} = K_i\left(\dfrac{\Psi\eta(1-\theta)}{I(t)} + 1\right) & \text{if } t > t_p \end{cases} \tag{4}$$

This switching characteristic is approximated by a smooth activation function denoted by $\varphi_a(t,t_p)$ is used, as presented in equation (5).