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On robustness of Lyapunov-based nonlinear adaptive controllers

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Abstract: In this paper, we discuss robustness of a class of control Lyapunov function (CLF)-based nonlinear adaptive controllers with respect to input uncertainties. We prove that the adaptive controllers are robust with respect to monotone input nonlinearities. Moreover, we extend this result to the robust set-point regulation problem of nonlinear systems. The robustness of the controllers also confirmed by computer simulations.

Keywords: nonlinear control, adaptive control, sector nonlinearity, monotone nonlinearity

1. INTRODUCTION

Set-point regulation of nonlinear systems, e.g. robot manipulator control, is a fundamental and important problem for control applications. Unlike standard regulation problems, an operating point is not a equilibrium point of open-loop system; this implies system parameter uncertainties directly cause an offset error.

To achieve offset-free set-point regulation, adaptive control is one of the effective approach. This fact is well known in the field of robot control (see, e.g. Craig (2005), Siciliano et al. (2010)), and many adaptive controllers for robot manipulators are proposed; Craig et. al. (1987), Slotine and Li (1987), Sadegh and Horowitz (1990), Tomei (1991), Berghuis et. al. (1993), and so on. Note that all of these controllers have the same structure; the combinations of an adaptive parameter compensation term and a stabilization term. Thanks to the adaptive compensation term, the effect of gravity is precisely canceled even if parameter uncertainties exist.

For general input affine nonlinear systems, the above construction of adaptive controllers could be extended by employing control Lyapunov functions (CLFs). In the set-point regulation problem, a CLF is also available as an adaptive control Lyapunov function (ACLF) (see Krstić et al. (1995), Satoh et al. (2009)). This implies both adaptive compensating and stabilizing terms can be designed based on the CLF.

As well-studied in the robot manipulator control, the adaptive controllers are robust with respect to parameter uncertainties. On the other hand, robustness with respect to input uncertainties such as gain margins or sector margins (Grad (1987), Sepulchre et al. (1997)) also important in practice. In Satoh et al. (2009), the authors showed that the CLF-based adaptive controllers have gain margins if the stabilization term itself have gain margins. However, robustness results with respect to more general input uncertainties are not studied.

In this paper, we discuss robustness of the CLF-based adaptive controllers with respect to a class of nonlinear input uncertainties. In particular, we consider the monotone nonlinearity (Arcak and Kokotović (2001), Fan and Arcak (2003)) as the input uncertainties and discuss the stability of the perturbed closed loop systems.

2. PRELIMINARIES

In this section, we introduce basic definitions of mathematical terms and their fundamental properties.

Let us consider the following nonlinear system:

$$\dot{x} = f(x) + g(x)u \tag{1}$$

where $x \in \mathbb{R}^n$ is the state and $u \in \mathbb{R}^m$ the control input. We assume $f : \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^n \to \mathbb{R}^{n\times m}$ are continuous mappings and $f(0) = 0$.

Control Lyapunov function (CLF) for (1) is defined as follows:

Definition 1. (control Lyapunov function). A C^1 function $V : \mathbb{R}^n \to \mathbb{R}$ is said to be a control Lyapunov function for (1) if the following properties holds:

(A1) V is proper; that is, the set $\{x \in \mathbb{R}^n | V(x) \le L\}$ is compact for every $L > 0$;
(A2) V is positive definite; that is, $V(0) = 0$ and $V(x) > 0$ for all $x \in \mathbb{R}^n \backslash \{0\}$;
(A3) the following holds:

$$\inf_{u\in\mathbb{R}^m} (L_f V + L_g V \cdot u) < 0,\ \forall x \in \mathbb{R}^n \backslash \{0\}, \tag{2}$$

where $L_f V$ and $L_g V$ are denote $(\partial V/\partial x)f(x)$ and $(\partial V/\partial x)g(x)$, respectively.

In this paper, we discuss the robustness of state feedback controllers with respect to input uncertainties. In nonlinear control theory, the following sector margins and gain

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margins are used to evaluate the robustness (Grad (1987), Sepulchre et al. (1997)):

Definition 2. (sector nonlinearity). A continuous mapping $\phi : \mathbb{R} \to \mathbb{R}$ is said to be a sector nonlinearity in $[\alpha, \beta]$, where $\alpha, \beta \in \mathbb{R}$ such that $0 < \alpha < 1 < \beta$, if the following conditions hold:

$$\begin{aligned} &\phi(0) = 0, \\ &\alpha u^2 \le u\phi(u) \le \beta u^2, \ \forall u \in \mathbb{R}\backslash\{0\}. \end{aligned} \tag{3}$$

Moreover, a mapping $\phi : \mathbb{R}^m \to \mathbb{R}^m; (u_1, \ldots, u_m)^T \mapsto (\phi_1(u_1), \ldots, \phi_m(u_m))^T$ is said to be a sector nonlinearity in $[\alpha, \beta]^m$ if each ϕ_i $(i = 1, \ldots, m)$ is a sector nonlinearity in $[\alpha, \beta]$.

Definition 3. (sector margin). Let $k : \mathbb{R}^n \to \mathbb{R}^m$ be a given state feedback controller and $\phi : \mathbb{R}^m \to \mathbb{R}^m$ any sector nonlinearity in $[\alpha, \beta]^m$. Then, the controller $u = k(x)$ is said to have a sector margin $[\alpha, \beta]^m$ if the origin of the following closed loop system is asymptotically stable:

$$\dot{x} = f(x) + g(x)\phi(k(x)). \tag{4}$$

Gain margins are also defined as a special case of sector margins:

Definition 4. (gain margin). The controller $u = k(x)$ is said to have a gain margin $[\alpha, \beta]^m$ if the condition of Definition 3 holds for any ϕ satisfying

$$\begin{aligned} &\phi(u) = Ku, \ \forall u \in \mathbb{R}^m, \\ &K = \mathrm{diag}(\kappa_1, \ldots, \kappa_m), \ \kappa_i \in [\alpha, \beta], \forall i \in \{1, \ldots, m\}. \end{aligned} \tag{5}$$

Moreover, such ϕ is called a gain uncertainty in $[\alpha, \beta]^m$.

Remark 1. Sector and gain margins contain the asymptotic stability of the original system (1). This follows from the fact that $\phi(u) = u$ is both sector nonlinearity and a gain uncertainty in $[\alpha, \beta]^m$.

3. GAIN MARGINS OF CLF-BASED ADAPTIVE CONTROLLERS

In this paper, we consider the following perturbed system of (1):

$$\dot{x} = f(x) + g(x)(u - \theta), \tag{6}$$

where $\theta \in \mathbb{R}^m$ is a constant parameter.

The problem considered here is the asymptotic stabilization of $x = 0$ of (6). As mentioned in section 5, this problem is closely related to non-zero set-point regulation of system (1).

To consider CLF-based controller design, we introduce the following hypothesis:

Hypothesis 1. There exists a CLF $V(x)$ for nominal system (1) (i.e. system (6) with $\theta = 0$).

Then it is natural to construct a stabilizing state feedback for system (6) by

$$u = k(x) + \theta, \tag{7}$$

where $k : \mathbb{R}^n \to \mathbb{R}^m$ asymptotically stabilizes the origin of (1) and guarantees the sector margin $[\alpha, \beta]^m$ for some α and β such that $0 < \alpha < 1 < \beta$. Note that Such $k(x)$ always exists under the Hypothesis 1. For example, we can employ Sontag's universal formula (Sontag (1989)) as $k(x)$.

The controller (7) clearly asymptotically stabilizes the origin of (6). However, by this construction, the sector margin of $k(x)$ is lost. More precisely, controller (7) does not guarantees any sector/gain margin for system (6) despite $k(x)$ guarantees the sector margin for (1).

To "recover" the robustness of $k(x)$, we extend the controller (7) to the following adaptive control form:

$$u = k(x) + \hat{\theta}, \tag{8}$$

$$\dot{\hat{\theta}} = -\Gamma L_g V^T, \tag{9}$$

where $\hat{\theta}$ is a estimate of θ, $\dot{\hat{\theta}}$ its update law, and $\Gamma := \mathrm{diag}(\gamma_1, \ldots, \gamma_m)$, $\gamma_i > 0$, $i \in \{1, \ldots, m\}$ a adaptive gain matrix.

Remark 2. V is available for adaptive control design since V is also a adaptive control Lyapunov function (ACLF) for (6). For details on ACLF, refer to Krstić et al. (1995).

Remark 3. The controller (8)–(9) is available whether θ is known or not.

Importantly, the controller (8)–(9) guarantees a gain margin for (6). The following theorem is the generalization of Lemma 2 in Satoh et al. (2009).

Theorem 2. Let $\phi(u) = Ku$ be any gain uncertainty in $[\alpha, \beta]^m$. Then the closed loop system

$$\begin{aligned} \dot{x} &= f(x) + g(x)\left[K\left(k(x) + \hat{\theta}\right) - \theta\right] \\ \dot{\hat{\theta}} &= -\Gamma L_g V^T \end{aligned} \tag{10}$$

is asymptotically stable at $(x, \hat{\theta}) = (0, K^{-1}\theta)$.

Proof. Let $\theta_2 := K^{-1}\theta$. Consider the following Lyapunov function $\tilde{V}$ for (10):

$$\tilde{V}(x, \hat{\theta} - \theta_2) := V(x) + \frac{1}{2}(\hat{\theta} - \theta_2)^T K\Gamma^{-1}(\hat{\theta} - \theta_2). \tag{11}$$

Then the time derivative of $\tilde{V}$ is obtained as

$$\begin{aligned} \dot{\tilde{V}} &= \frac{\partial V}{\partial x}\left[f(x) + g(x)\left(K(k(x) + \hat{\theta}) - \theta\right)\right] \\ &\quad + (\hat{\theta} - \theta_2)^T K\Gamma^{-1}\dot{\hat{\theta}} \\ &= \frac{\partial V}{\partial x}\left(f(x) + g(x)Kk(x)\right) \\ &\quad + L_g V K\left(\hat{\theta} - \theta_2\right) - (\hat{\theta} - \theta_2)^T K\Gamma^{-1}\left(\Gamma L_g V^T\right) \\ &= \frac{\partial V}{\partial x}\left(f(x) + g(x)Kk(x)\right) \le 0. \end{aligned}$$

Note that the last inequality holds since $k(x)$ guarantees the gain margin. The convergence of x and $\hat{\theta}$ follows from the LaSalle's invariance principle (for more details, see the proof of Theorem 5 in section 4).

This theorem provides that the controller (8)–(9) is robust to gain uncertainties in $[\alpha, \beta]^m$. Is it possible to extend this result to more general input uncertainties ? We tackle this problem in the following section.

4. ROBUSTNESS WITH RESPECT TO MONOTONE UNCERTAINTIES

4.1 Monotone Nonlinearities

In Theorem 2, the key of the proof is that any gain uncertainty ϕ satisfies

$$\phi(k(x) + \hat{\theta}) = \phi(k(x)) + \phi(\hat{\theta}). \tag{12}$$