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Static Smooth Control Lyapunov Function Design for differentially Flat Systems

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Abstract: We propose a C^{∞} differentiable strict CLF design method for differentially flat systems by dynamic extension and minimum projection method.

Keywords: Nonlinear control, Control Lyapunov function.

1. INTRODUCTION

Every stabilizable control system on Euclidean space attains a control Lyapunov function (CLF) (Clarke et al. (1997)), although CLF design for nonlinear control systems is often a difficult problem.

Isidori (1986) focused that some nonlinear control systems can be linearized by state and input transformations with dynamic compensators. For differentially flat systems (Fliess et al. (1994)), as one of such systems, we can easily design quasi static CLFs with dynamic compensators. However these quasi static CLFs based controllers need dynamic input transformations and cause delays (Kuga et al. (2015)), singularity points (Hua et al. (2013)) or lose robustness (Hauser et al. (1992)).

Freeman et al. (1996) showed that for every static state feedback linearizable nonlinear control system with a static feedback law, we can easily design a static CLF. However, many nonlinear control systems can not be exact linearized, and to achieve a static CLF is often a difficult problem.

By the investigation of Levine (2009), most of all controllable systems of trajectory generation, motion planning, or tracking are differentially flat systems. By attentions of vertical take off and landing drones or automatic operating car systems, differentially flat systems as trajectory problems attain many interests.(Fliess et al. (1994), Notarstefano et al. (2005), Hua et al. (2013))

A non-smooth CLF design method for nonlinear control systems by the minimum projection method was proposed by Yamazaki et al. (2013). In this paper, we establish a static smooth CLF design method for differentially flat systems by the minimum projection method. Static smooth CLF enables us to design static state feedback controllers which can avoid delays, singularities points or losing robustness.

[1]

The CLF is designed by the following two steps. First, we design a quasi static CLF with dynamic compensators for the linearized augmented system. Then we apply the minimum projection method to the obtained quasi static CLF and design a static C^{∞} differentiable CLF for the system. Our proposed CLF is a smooth function. This means that we can design inverse optimal controllers such as Sontag type controllers (Sontag (1989)) for the differentially flat systems.

Finally, we show computer simulation to confirm that our proposed static smooth CLF based controller can asymptotically tracks a desired trajectory for a two wheeled robot.

2. PRELIMINARIES

We introduce basic definitions and properties used in the paper.

2.1 Nonlinear Control System

Throughout the paper, we consider the following nonlinear control system (Rosier et al. (2013)):

$$\dot{x} = f(x) + g(x)u, \tag{1}$$

where $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$ denote a state and an input respectively. Mappings $f : \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^n \to \mathbb{R}^{n\times m}$ are supposed to be locally Lipschitz continuous with respect to both x and u, and satisfy $f(0,0) = 0$.

2.2 Differentially Flat System

In this paper we consider CLF design method for a differentially flat system. In accordance with Fliess et al. (1994), we introduce the differentially flat system as follows.

Definition 1. Consider system (1), and the following dynamic compensator:

$$\begin{bmatrix} \dot{p} \\ u \end{bmatrix} = \begin{bmatrix} a(x,p,v) \\ b(x,p,v) \end{bmatrix} \qquad (p \in \mathbb{R}^l, v \in \mathbb{R}^m), \tag{2}$$

where p and v denote a state and an input of the dynamic compensator, respectively.

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With the dynamic compensator (2), we also considers the following augmented system of (1) on extended state space $\mathbb{R}^{n+l}$:

$$\begin{bmatrix} \dot{x} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} f(x) + g(x)b(x,p,v) \\ a(x,p,v) \end{bmatrix}, \tag{3}$$

where the origin is $(x,p) = (0,0)$. Finally the augmented system becomes a linear control system $\phi \in \mathbb{R}^{n+l}$:

$$\dot{\phi} = A\phi + Bv, \tag{4}$$

with a following diffeomorphism $\Phi : \mathbb{R}^{n+l} \to \mathbb{R}^{n+l}$:

$$\phi = \Phi(x,p), \quad \Phi(0) = 0, \tag{5}$$

where the matrix pair (A, B) is controllable. The system (1) which is linearizable via such diffeomorphism with dynamic compensator is said to be a differentially flat system.

2.3 Control Lyapunov Function (CLF)

In this paper we design a static smooth CLF (Artstein (1983)) as below.

Definition 2. (Control Lyapunov function (CLF)). Consider system (1). A proper positive definite C^∞ differentiable function $V : \mathbb{R}^n \to \mathbb{R}$ satisfying the following condition is said to be a static smooth control Lyapunov function (CLF) for (1):

$$L_f V(x) < 0, \ \forall x \in \{x \in X | L_g V = 0\}, \tag{6}$$

where $L_f V = (\partial V / \partial x) f(x)$ and $L_g V = (\partial V / \partial x) g(x)$. Then $\dot{V}(x,u)$ can be written as follows:

$$\dot{V}(x,u) = L_f V(x) + L_g V(x) u. \tag{7}$$

2.4 Static Smooth CLF Design for Static State Feedback Linearizable Systems

For static state feedback linearizable nonlinear control systems, a static smooth CLF can be easily designed (Freeman et al. (1996)).

Proposition 1. If there exists a diffeomorphism $\varphi = \Phi(x)$ with $\Phi(0) = 0$ that transforms system (1) into

$$\dot{\varphi} = A\varphi + B\left[l_0(\varphi) + l_1(\varphi)u\right], \tag{8}$$

where, the matrix pair (A, B) is controllable and mappings $l_0 : \mathbb{R}^n \to \mathbb{R}^n$ and $l_1 : \mathbb{R}^n \to \mathbb{R}^{n\times m}$ are continuous, and $l_1(\Phi)$ is nonsingular for all Φ, the system is said to be a static state feedback linearizable systems.

For the static state feedback linearizable system we can easily design a static smooth CLF as follows.

Proposition 2. Let P be a symmetric positive definite solution of the following Riccati equation for the system (8):

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{9}$$

for arbitrary positive definite matrices Q and R.

Then, function $\tilde{V}(x) : \mathbb{R}^n \to \mathbb{R}$ defined by the following function is an CLF for the system:

$$\tilde{V}(\varphi) = \varphi^T P \varphi. \tag{10}$$

Moreover function $\tilde{V}(\Phi(x)) : \mathbb{R}^n \to \mathbb{R}$ defined by the following function is also a CLF for $\Phi(x)$:

$$V(x) = \Phi^T(x) P \Phi(x). \tag{11}$$

2.5 Sontag type controller

Sontag (1989) proposed a smooth static state feedback controller with a static smooth CLF.

Proposition 3. Let $V : \mathbb{R}^n \to \mathbb{R}$ a CLF for system (1). Then, the following input $u : \mathbb{R}^n \to \mathbb{R}^m$ asymptotically stabilizes the origin of the system (1):

$$u(x) = \begin{cases} -\dfrac{a + \sqrt{a^2 + b^2}}{b} & (b \neq 0) \\ 0 & (b = 0), \end{cases} \tag{12}$$

where $a = L_f V$ and $b = L_g V$.

2.6 Inverse Optimal Control with CLF

Nakamura et al. (2013) proposed an inverse optimal controller with a static smooth CLF.

Proposition 4. Let $a_j > 0$ be a constant for all $j \in \{1, \ldots, m\}$, $V : \mathbb{R}^n \to \mathbb{R}$ a CLF for systems (1). Then, we define γ as follows:

$$\gamma(x) = \sum_{j=1}^{m} \frac{1}{a_j + 1} \cdot \frac{1}{R(x)} |L_{g_j} V(x)|^{a_j + 1} - L_f V(x), \tag{13}$$

where, $R : \mathbb{R}^n \to \mathbb{R}_{>0}$ is a positive-valued function and $\gamma(x)$ is a positive definite function. Then, the following input $u_j : \mathbb{R}^n \to \mathbb{R}^m$ asymptotically stabilizes the origin of the system (1):

$$u_j(x) = -\frac{1}{R(x)} |L_{g_j} V(x)|^{a_j} sgn(L_{g_j} V(x)), \quad (j = 1, \ldots, m), \tag{14}$$

and minimizes the following cost function:

$$J = \int_0^\infty \left[\gamma(x) + \sum_{j=1}^{m} \frac{a_j}{a_j + 1} R^{1/a_j}(x) |u_j|^{(a_j+1)/a_j} \right] dt. \tag{15}$$

Further the input is continuous and achieves at least a gain margin $[1/(\min_{1\le j\le m} a_j + 1), \infty)$.

3. PROBLEM STATEMENT

In this paper we propose a static smooth CLF asymptotically stabilizes the origin of the differentially flat system (1) satisfying the following assumption.

Assumption 1. Diffeomorphism Φ in (5) can be written as follows:

$$\Phi(x,p) = \alpha(x)p + \beta(x) \tag{16}$$

where, $\alpha : \mathbb{R}^n \to \mathbb{R}^{(n+l)\times l}$ and $\beta : \mathbb{R}^n \to \mathbb{R}^(n+l)$ are C^∞ differentiable mappings, and α is full column rank.

Remark 1. Many differentially flat systems satisfy the above assumption (See examples in Levine (2009).). However we have not found the exact condition. Thus we suppose the above condition.

In this paper we design a static state feedback control law introduced by Isidori (1995).

Definition 3. If the value of the control at time t depends only on the values, at the same instant of time, of the state x and of the external reference input, the control is said to be a Static State Feedback Control.