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Explicit Model Predictive Control via Nonlinear Piecewise Approximations ⋆

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Abstract: In this paper, a novel identification methodology is proposed to capture general multivariate nonlinear relationships, with focus on the bounded-error approximation of model predictive control for constrained (non)linear systems. The formulation of the identification problem takes, at each iteration, the form of a constrained linear (or quadratic) optimization problem that is mathematically feasible as well as numerically tractable. The efficiency of the proposed method for the derivation of low-complexity explicit model predictive controllers is demonstrated via the constrained control of a thermodynamic power plant.

Keywords: System Identification, Nonlinear Approximation, Explicit Model Predictive Control.

1. INTRODUCTION

Model Predictive Control (MPC, see [Mayne et al. (2000)]) is a widely used control method which involves the solution at each sampling instant of a finite horizon optimal control problem subject to the system dynamics and input and state constraints. Nevertheless, the on-line solution of an optimization problem is often time consuming and requires expensive computation. This makes the real-time MPC implementation usually limited to processes where the sampling time and hardware capability are sufficient to support the computational needs.

To overcome the aforementioned implementation issues, several approaches to explicit MPC (EMPC) have been recently pursued, where the on-line computation would simply be point location search and function evaluation. An interesting survey on this topic can be found in [Alessio and Bemporad (2009)]. In the context of linear systems, the classical problem of obtaining EMPC controllers with quadratic cost and linear constraints can be solved by using parametric quadratic programming techniques (see, for instance, [Bemporad et al. (2002)]). As for nonlinear systems, deriving the true optimal nonlinear MPC (NMPC) control law is generally not possible and hence approximate approaches have to be followed. Following this direction, several techniques have been devised such as artificial neural networks [Pin et al. (2013)], set memberships identification [Canale et al. (2009), Fagiano et al. (2012)] and piecewise affine (or linear) approximators [Johansen (2004), Grancharova and Johansen (2012), Grancharova and Olaru (2014)]. Nevertheless, while piecewise affine approaches normally suffer from the "curse of dimensionality", standard nonlinear approximators offer universal capabilities at the price of non-convex optimization schemes. Hence, the investigation of EMPCs with nonlinear piecewise representations is of interests.

In this paper, we introduce an extension of the identification method recently proposed in [Alamir (2013)]. This method is suitable for the component-wise approximation of MPC control laws, i.e. each control input will be identified independently. The obtained approximate EMPC is represented as piecewise nonlinear approximators, allowing to reduce the number of regions with respect to the piecewise affine approaches. Compared to the neural networks approach, the methodology takes advantages of efficient computation of constrained linear (or quadratic) programming problems. Bounds on admissible errors are also given in order to maintain the closed-loop performance as well as to trade off complexity and approximation error.

This paper is organized as follows. In Section 2, the design of a nonlinear piecewise approximator is presented. Section 3 describes a heuristic procedure for approximating general nonlinear functions as well as MPC control laws based on the designed approximator. Simulation results are reported in Section 4 before drawing the conclusions in the final section.

Notation

- $\mathbb{I}_{a:b} := \{a, a+1, \ldots, b-1, b\}$, $a, b \in \mathbb{N}$, $a < b$;
- p_i, $i \in \mathbb{I}_{1:n}$: components of a vector $p \in \mathbb{R}^n$;
- $\text{card}(\mathcal{S})$: the cardinality of the set $\mathcal{S}$, i.e. the number of elements of the set;
- $N(P)$: the neighborhood of a point, a set or a topological space P within an appropriate tolerance;
- ∂P: the boundary of a topological space P.

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2. DESIGN OF A NONLINEAR APPROXIMATOR

2.1 Problem statement and mathematical formulation

Consider the following nonlinear identification problem:
Problem 1. *Given the data* $\mathcal{D} = \{(q(k), Z(k)\}_{k\in\mathbb{I}_{1:N}}$ *where* $Z \in \mathcal{H} \subseteq \mathbb{R}^{n_z}$ *is the regressor and* $q \in [\underline{q}, \overline{q}] \subset \mathbb{R}$ *is the output. Find a map* $F : \mathbb{R}^{n_z} \to \mathbb{R}$ *of the form:*

$$F(Z) := \Gamma^{-1}(L^T Z);\ \Gamma(\cdot) \text{ is strictly increasing} \quad (1)$$

such that the following approximation holds:

$$q \approx F(Z) \quad (2)$$

The structure (1) is a Wiener-like model with nonlinear piecewise strictly monotonic static output mapping which can be parametrized using a finite function basis:

$$\Gamma(q) = \sum_{j=1}^{n_b} [B^{(j)}(\xi(q))]\mu_j = B(\xi(q))\mu;\ \xi(q) = \frac{q-\underline{q}}{\overline{q}-\underline{q}} \quad (3)$$

where the basis functions are given by:

$$\{B^{(j)}\}_{j\in\mathbb{I}_{1:n_b}} := \{1\} \cup \{B_1^{(i)}\}_{i\in\mathbb{I}_{2:n_m}} \cup \{B_2^{(i)}\}_{i\in\mathbb{I}_{1:n_m}} \quad (4)$$

The number of functions is $n_b = 2n_m$ while $B_1^{(i)}$ and $B_2^{(i)}$ are defined as:

$$B_1^{(i)}(\eta) := (1+\alpha_i)\frac{\eta}{1+\alpha_i\eta};\ B_2^{(i)}(\eta) := \frac{\eta}{1+\alpha_i(1-\eta)} \quad (5)$$

The coefficients α_i are given by $\alpha_i := e^{\beta(i-1)} - 1$ for some constant $\beta > 0$ (see [Alamir (2013)]).

Denote $\mu \in \mathbb{R}^{n_b}$ and $L \in \mathbb{R}^{n_z}$ as the parameters of $F(Z)$. The total number of parameters is $n_p = n_b + n_z$. The basic idea of the above formulation is to solve Problem 1 by finding $\mu \in \mathbb{R}^{n_b}$ and $L \in \mathbb{R}^{n_z}$ such that the approximation $B(\xi(q))\mu \approx L^T Z$ holds. Hence, consider the following linear program (LP) with a positive constant ϵ:

$$\begin{aligned} \min_{\mu,L} \quad & \max_{(q,Z)\in\mathcal{D}} |w(q,Z)\cdot(B(q)\mu - Z^T L)| \\ \text{s.t.} \quad & [\frac{dB}{d\xi}(\xi)]\mu \geq \epsilon,\ \forall \xi \in [0,1] \end{aligned} \quad (6)$$

where the indicator $w(q,Z)$: $\mathbb{R}\times\mathbb{R}^{n_z} \to \mathbb{R}_+$ is introduced in (6) to enforce specific precision depending on the problem. We define this weight as a function of the learning data:

$$w(q,Z) = \begin{cases} \rho_i & \text{if } (q,Z) \in \mathcal{W}^{(i)} \\ 1 & \text{otherwise} \end{cases} \quad (7)$$

where ρ_i are some positive constants and $\mathcal{W}^{(i)}$ are disjoint subspaces. This formulation obviously recalls the known *weighted norm approximation* where ∞-norm is employed. Alternative formulation, based on the L_2-norm, can also be adopted leading to a quadratic programming (QP) problem.

The constraint expresses the fact that Γ has to be strictly monotonic in order to guarantee the existence of the inverse map $\Gamma^{-1}(\cdot)$. Note that by defining a sufficiently dense grid of ξ over the interval $[0,1]$, namely $\{\xi_i\}_{i\in\mathbb{I}_{1:n_\xi}}$, this constraint can be transformed into a finite number of linear inequalities.

Remark 1. The normalization constraint in the formulation of [Alamir (2013)] has been removed as the imposed constraint guarantees that $\mu \neq 0$, i.e. the trivial solution ($\mu = 0$, $L = 0$) is never admissible.

2.2 Preliminary analysis

For convenience, we denote $p_{id} = (n_m, \beta, \epsilon, \{\xi_i\}_{i\in\mathbb{I}_{1:n_\xi}}, w(\cdot))$ as *identification parameters* and *model parameters* as $p = \begin{bmatrix}\mu \\ L\end{bmatrix} \in \mathbb{R}^{n_p}$. Let the slack variable ζ be the minimizing cost, the LP (6) is rewritten as follows

$$\begin{aligned} \mathcal{L}_{(p_{id},\mathcal{D})}(p,\zeta):\ & \min_{p,\zeta} \quad \zeta \\ & \text{s.t.} \quad A(p_{id},\mathcal{D})\cdot\begin{bmatrix}p\\ \zeta\end{bmatrix} \leq b(p_{id},\mathcal{D}) \end{aligned} \quad (8)$$

where $A(p_{id},\mathcal{D})$ and $b(p_{id},\mathcal{D})$ are easily derived.

The feasibility of $\mathcal{L}_{(p_{id},\mathcal{D})}(p,\zeta)$ is as follows:
Proposition 1. *The LP* $\mathcal{L}_{(p_{id},\mathcal{D})}(\cdot)$ *is feasible.*

Proof. According to the definition of the functional basis, $[\frac{dB}{d\xi}(\xi)] = \left[\frac{dB^{(j)}}{d\xi}(\xi_i)\right]_{i\in\mathbb{I}_{1:n_\xi}, j\in\mathbb{I}_{1:n_b}}$ is an $n_\xi \times n_b$ matrix with the elements of the first column being zeros while the remaining being positive. Hence, it is obvious that there exists μ such that $[\frac{dB}{d\xi}(\xi)]\mu > 0$. Thus, the feasibility of $\mathcal{L}_{(p_{id},\mathcal{D})}(\cdot)$ is guaranteed with any $\epsilon > 0$. □

The identification residual can be characterized as follows:
Proposition 2. *If* (μ, L) *is a feasible solution of the LP* $\mathcal{L}_{(p_{id},\mathcal{D})}(\cdot)$ *with corresponding cost* J*, the identification residual of any learning data point* (q, Z) *is such that:*

$$|q - F(Z)| \leq \frac{1}{w(q,Z)}\cdot\frac{1}{\epsilon}\cdot(\overline{q}-\underline{q})\cdot J \quad (9)$$

Proof. For any $(q,Z) \in \mathcal{D}$, the continuity of $\Gamma(\cdot)$ implies the existence of $\hat{q} = F(Z)$ such that $B(\xi(\hat{q}))\mu = L^T Z$. One clearly has

$$\begin{aligned} J &\geq w(q,Z)|B(\xi(q))\mu - L^T Z| \\ &= w(q,Z)|B(\xi(q))\mu - B(\xi(\hat{q}))\mu| \\ &\geq \min_{\xi\in[0,1]}\left[(\frac{dB}{d\xi}(\xi)\mu)\right] w(q,Z)|\xi(\hat{q}) - \xi(q)| \\ &\geq \epsilon w(q,Z)|\xi(\hat{q}) - \xi(q)| = \epsilon w(q,Z)\frac{|\hat{q}-q|}{\overline{q}-\underline{q}} \end{aligned} \quad (10)$$

which is equivalent to (9). □

Remark 2. Proposition 2 implies that the desired fit can be obtained with a sufficiently small cost and appropriate identification parameters. It is also possible to adjust the relative fit between subsets of data through $w(q,Z)$. Nevertheless, the upper bound (9) on the identification residual can be very conservative in some cases.

3. NONLINEAR PIECEWISE APPROXIMATIONS AND MODEL PREDICTIVE CONTROL

3.1 Problem statement and proposed methodology

The lack of universal property of the proposed nonlinear approximator (1) has been underlined in [Alamir (2013)]. In order to overcome this structural limitation, we consider the following identification problem: