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Robust Model Predictive Control of Systems by Modeling Mismatched Uncertainty

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Abstract: This study addresses to the robustness of model predictive control in the presence of the mismatched uncertainty, e.g. disturbance, noise and parameter variations. Model predictive control is solved online and its control action is fed to the real system with the additional control action that is required to maintain the controlled trajectories in a simple uncertainty tube in practice where the center of the aforementioned tube is the trajectory of the nominal model. For this purpose, a sliding mode controller as variable control structure is designed taking the difference between the real system and nominal system into consideration. The stability of the overall system is proven taking the modeling error on the uncertainty model into account.

Keywords: Mismatched uncertainty, model predictive control, robustness, sliding mode control, tubes, uncertain systems.

1. INTRODUCTION

Model predictive control (MPC), an extension of optimal control, is a model-based control approach which is able to deal with the constraints on the states and inputs (Mayne et al., 2000; Kayacan et al., 2014). The concept of MPC is to compute state trajectories in a finite horizon by minimizing a cost function consisting of all states and inputs online (Kayacan et al., 2015c). After solving optimization problem, only the first element of the generated input sequence is fed to the system (Kayacan et al., 2015a). Then, the finite horizon is shifted over time for the next sampling time (Morari and Lee, 1999; Qin and Badgwell, 2003). The same procedure is repeated again when new measurements or estimates are obtained.

The robust stability of MPC is obtained only if the nominal model is inherently robust without estimation errors, uncertainties and parameter variations. However, systems are always subjected to disturbances in real life (Grimm et al., 2004). Inasmuch as unmodeled uncertainties may result in instability of systems, robust MPC method has become a crucial topic and been developed due to the emphasis of the handling uncertainties in practice (Calafiore and Fagiano, 2013; Yan and Wang, 2014). One of the most significant method is the tube-based approach proposed for state and output feedback MPC (Langson et al., 2004; Mayne et al., 2005, 2006). In these previous studies, the uncertainty has not been modeled and the additional control action is designed in which the uncertainty error is multiplied by a state feedback controller and then the product is fed to the real system. Furthermore, an integral sliding manifold (ISM) has been proposed instead of a state feedback controller (Rubagotti et al., 2011).

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This paper focuses on robust model predictive control for systems with mismatched uncertainties. The main contribution of this study is to formulate a tube-based approach by modeling the uncertainty structure. In order to handle the uncertainties in the real system, sliding mode control method, which is inherently robust to uncertainties, is used to design a controller by taking the difference between the nominal model and real system. The stability of the overall system is proven by using a Lyapunov function.

This paper is organized as follows: The previous works are summarized in Section 2. The controller is designed and the control structure is presented in Section 3. The system is presented in Section 4. The simulation results are given in Section 5. Finally, a brief conclusion is given in Section 6.

2. PROBLEM FORMULATION

The discrete-time linear time-invariant system model is represented by

$$x_{k+1} = Ax_k + Bu_k + w_k \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the control vector and $w \in \mathbb{R}^n$ is the disturbance vector. The constraints on the state and input are denoted by

$$x \in \mathbb{X}, \quad u \in \mathbb{U}, \tag{2}$$

where $\mathbb{X} \subset \mathbb{R}^n$ is closed, $\mathbb{U} \subset \mathbb{R}^m$ is compact and they have their own origin in their own interior. It is assumed that the disturbance w is bounded

$$w \in W \tag{3}$$

where W is compact and includes the origin.

The nominal system respecting the system (1) is denoted by

$$\bar{x}_{k+1} = A\bar{x}_k + B\bar{u}_k \tag{4}$$

It is assumed that the controller $\bar{u}_k$ is equal to $K\bar{x}_k$ where $K \in \mathbb{R}^{m\times n}$ denotes the coefficients of the controller and the closed-loop system $A^K = A + BK$ is stable. The disturbance set denoted

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by Z for the real system model $x_{k+1} = A^K x_k + w$ fulfills the following condition,

$$A^K X \oplus W \subseteq Z \tag{5}$$

where $\oplus$ is Minkowski set addition, the disturbance set Z is invariant and the origin of the real system.

In order to control the real system with mismatched uncertainty in (1), a tube-based MPC approach was proposed in (Mayne and Langson, 2001) as below.

Proposition 1. It is presumed that $x \in \bar{x} \oplus Z$ where Z is the invariant disturbance for $x_{k+1} = Ax_k + Bu_k + w$ and $\bar{x}_{k+1} = A\bar{x}_k + B\bar{u}_k$. If the control input to the real system $u(\bar{u},x,\bar{x}) = \bar{u}(\bar{x}) + K(x-\bar{x})$, then $x_{k+1} \in \bar{x}_{k+1} \oplus Z \; \forall w \in W$.

This proposition clearly expresses that the feedback controller $u(\bar{u},x,\bar{x}) = \bar{u}(\bar{x}) + K(x-\bar{x})$ enforces the states x of the real system $x_{k+1} = A^K x_k + w$ to track the states $\bar{x}$ of the nominal system $\bar{x}_{k+1} = A\bar{x}_k + B\bar{u}_k$. This proposed control method has been called the tube-based approach. In this former tube-based MPC approach, the states of the nominal model are directly fed to MPC and thus MPC does not have any information exchange with the real system (Langson et al., 2004). This structure was criticized due to the fact that it does not take the measurements coming from the real system into consideration. In order to interact MPC with the real-system, a new method that not only MPC but also the nominal model is initialized by the measurements of the real system at each sampling time instant has been proposed for state feedback case (Mayne et al., 2005) as in the following proposition.

Proposition 2. It is presumed that $x \in \bar{x} \oplus Z$ where Z is the invariant disturbance for $x_{k+1} = Ax_k + Bu_k + w$ and $\bar{x}_{k+1} = A\bar{x}_k + B\bar{u}_k$. If the control input to the real system $u(\bar{u},x,\bar{x}) = \bar{u}(x) + K(x-\bar{x}(x))$, then $x_{k+1} \in \bar{x}_{k+1} \oplus Z \; \forall w \in W$.

As an alternative method, only initialization of MPC by the measurements of the real system has been proposed in (Kayacan et al., 2015b, 2016). In these studies, all states of the real system are assumed to be measurable. However, this is not feasible in practice since number of sensors are generally less than number of measured variables. For this reason, output feedback MPC has been developed while a Luenberger observer is employed to estimate immeasurable states (Mayne et al., 2006).

In addition to a state feedback controller, an integral sliding manifold (ISM) has been proposed to handle uncertainties (Rubagotti et al., 2011) as in Proposition 3. In this approach, measurements coming from the real system are fed to MPC while the nominal model is not initialized by measurements coming from the real system.

Proposition 3. It is presumed that $x \in \bar{x} \oplus Z$ where Z is the invariant disturbance for $x_{k+1} = Ax_k + Bu_k + w$ and $\bar{x}_{k+1} = A\bar{x}_k + B\bar{u}_k$. If the control input to the real system $u(\bar{u},x,\bar{x}) = \bar{u}(x) - ksgn(x-\bar{x})$, then $x_{k+1} \in \bar{x}_{k+1} \oplus Z \; \forall w \in W$.

In these approaches, the stability analysis is proven over the real system due to the unmodeled uncertainty. In this paper, the nonlinear modeling of the uncertainty model is formulated and then the stability analysis is proven over this uncertainty model as distinct from the previous ones.

3. ROBUST MODEL PREDICTIVE CONTROL

In the controller design process, it is assumed that the nominal control input generated by MPC $\bar{u} = Kx$ is able to stabilize the nominal system so that $A + BK$ is stable. Since the real system and the nominal system are not identical, a controller is required to stabilize the uncertainty model defined as the difference between the real system and nominal model. The uncertainty state z is formulated as

$$z = x - \bar{x}(x) \tag{6}$$

where x and $\bar{x}$ are the states of the real system and nominal system.

Proposition 4. It is presumed that $x \in \bar{x} \oplus Z$ where Z is the invariant disturbance for $x_{k+1} = Ax_k + Bu_k + w$ and $\bar{x}_{k+1} = A\bar{x}_k + B\bar{u}_k$. The uncertainty model is described as a second-order nonlinear model $\ddot{z} = h(z) + v$ where $z = x - \bar{x}(x)$ is the output of the uncertainty model while v is the input of the uncertainty model. If the control input to the real system $u = \bar{u}(x) + v(x,\bar{x}(x))$, then $x_{k+1} \in \bar{x}_{k+1} \oplus Z \; \forall w \in W$.

The feedback control action $v(x,\bar{x}(x))$ in aforementioned proposition, which requires low sensitivity to plant parameter uncertainty and finite-time convergence, will be formulated based on sliding mode control (SMC) theory. The proposed control structure is illustrated in Fig. 1.

The uncertainty model is represented by the second-order nonlinear model as follows:

$$\ddot{z} = h(z) + v \tag{7}$$

where v is the control input, z is the output of the system and $h(z)$ is the nonlinear or time-varying dynamics of the system. The system output z is measurable while the system dynamics $h(z)$ is not known. It is assumed that the function h is upper bound by H as follows:

$$| h | \leq H \tag{8}$$

The tracking error is written as

$$\tilde{z} = z - z_d \tag{9}$$

where $\tilde{z}$ is the tracking error and z_d is the desired trajectory of the system.

A sliding surface is defined to track the desired trajectory of the system as follows:

$$s = \dot{\tilde{z}} + \lambda \tilde{z} \tag{10}$$

where λ is a positive constant and denotes the slope of the sliding surface. As can be seen in (10), the sliding surface

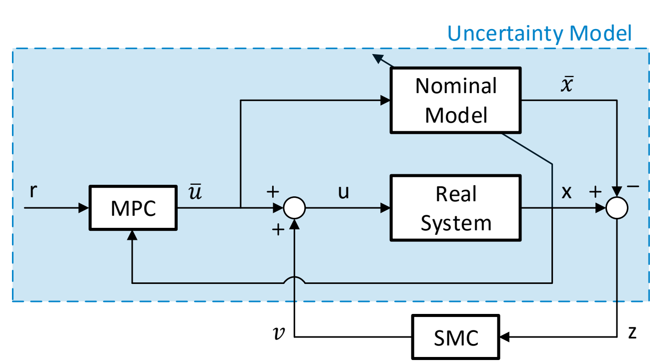

Fig. 1. Control Scheme