Available online at www.sciencedirect.com
ScienceDirect
IFAC PapersOnline CONFERENCE PAPER ARCHIVE
IFAC-PapersOnLine 49-18 (2016) 270–275

Stochastic Nonlinear Model Predictive Control with Joint Chance Constraints

Vinay A. Bavdekar * **Ali Mesbah** *

* *Department of Chemical and Biomolecular Engineering, University of California, Berkeley, CA 94720 USA*
e-mail: {bavdekar,mesbah}@berkeley.edu

Abstract: When the stochastic description of system uncertainties is available, a natural approach to predictive control of uncertain systems involves explicitly accounting for the probabilistic occurrence of uncertainties in the optimal control problem. This work presents a stochastic nonlinear model predictive control (SNMPC) approach for nonlinear systems subject to time-invariant uncertainties as well as additive disturbances. The generalized polynomial chaos (gPC) framework is used to derive a deterministic surrogate for the stochastic optimal control problem. The key contribution of this paper lies in extending the gPC-based SNMPC approach reported in our earlier work to handle stochastic disturbances. This is done via mapping the stochastic disturbances onto the space of the coefficients of polynomial chaos expansions, which enables efficient propagation of stochastic disturbances. A sample-based approach to joint chance constraint handling is employed to fulfill the state constraints in a probabilistic sense. A gPC-based Bayesian parameter estimator is utilized to update the probability distribution of uncertain system parameters at each sampling time. In a simulation case study, the closed-loop performance of the SNMPC approach is demonstrated on an atmospheric-pressure plasma jet that is developed for biomedical applications.

Keywords: Stochastic optimal control, Chance constraints, Generalized polynomial chaos, Nonthermal atmospheric-pressure plasmas

1. INTRODUCTION

Predictive control strategies are widely used for advanced control of complex systems owing to their ability to deal with multivariable system dynamics, constraints, and competing control objectives (Morari and Lee, 1999). Even though model predictive control (MPC) exhibits a certain degree of robustness to system uncertainties due to its receding-horizon implementation, the deterministic framework of MPC cannot systematically handle uncertainties (Mayne, 2014). Substantial work has been done in the area of robust MPC with the aim of accounting for system uncertainties in the optimal control problem. Robust MPC generally relies on set-membership uncertainty descriptions (i.e., bounded, deterministic descriptions), and requires constraints be satisfied with respect to all uncertainty realizations (Bemporad and Morari, 1999).

In many practical control applications, however, system uncertainties are of stochastic nature. When probabilistic descriptions of uncertainties can be characterized, a natural approach to MPC involves explicitly considering the stochastic occurrence of system uncertainties in the optimal control problem. Stochastic MPC (SMPC) with chance constraints provides a systematic framework for optimal control of stochastic systems. Chance constraints, which constitute a key component of SMPC, allow for ensuring an admissible level (in a probabilistic sense) of robustness to uncertainties in constraint handling. Effective constraint handling in a stochastic setting is critical to MPC of uncertain systems, in particular when high-performance operation is realized in the vicinity of constraints.

Recent years have witnessed significant advances in SMPC for linear systems (see (Mesbah, 2016) for an overview of various SMPC formulations). However, stochastic optimal control of nonlinear systems has received relatively little attention mainly due to the challenges associated with efficient uncertainty propagation and establishing the closed-loop properties of SMPC in a nonlinear setting. van Hessem and Bosgra (2006) presented a stochastic nonlinear MPC (SNMPC) approach based on optimizing a deterministic feedforward trajectory and a linear time-varying feedback controller for, respectively, constraint handling and minimizing the closed-loop variance around a reference trajectory. Markov chain and sequential Monte Carlo techniques were employed in (Lecchini-Visintini et al., 2006; Kantas et al., 2009) to develop sample-based approaches to solving stochastic nonlinear optimal control problems. The computational complexity of these approaches, however, prevented their receding-horizon implementation.

The generalized polynomial chaos (gPC) framework (Xiu and Karniadakis, 2002) was used in (Fagiano and Khammash, 2012; Mesbah et al., 2014) to develop SNMPC approaches for nonlinear systems subject to time-invariant probabilistic uncertainties in parameters and initial conditions. In the gPC framework, each stochastic system state is approximated by an expansion of orthogonal polynomial basis functions, defined based on the known descriptions of probabilistic uncertainties. The polynomial

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2016.10.176

chaos expansions of states provide an efficient machinery for uncertainty propagation. The statistical moments of stochastic states can be efficiently computed from the expansion coefficients or, alternatively, the expansions can be used as a surrogate for the nonlinear system model to perform Monte Carlo simulations efficiently. In (Mesbah et al., 2014), the moments of stochastic states are used to replace individual chance constraints with deterministic approximations. These gPC-based SNMPC approaches, however, cannot account for the effect of stochastic disturbances. For nonlinear control-affine systems with additive stochastic disturbances, Buehler et al. (2016) have recently presented a SNMPC approach that uses the Fokker-Planck equation for describing the evolution of probability distributions of states.

This paper addresses the MPC problem for stochastic nonlinear systems subject to uncertain initial conditions, parametric uncertainties, and additive white noise processes. The uncertain initial conditions and system parameters can be described by arbitrary probability distributions with a finite variance. The gPC framework is employed for uncertainty propagation. A key challenge in using the gPC framework for uncertainty propagation lies in efficient handling of stochastic disturbances since a large number of basis functions is often required in polynomial chaos expansions to adequately describe time-varying uncertainties. The main contribution of this work is to account for the effect of stochastic disturbances in a gPC-based SNMPC approach with joint chance constraints. The Galerkin projection method (Ghanem and Spanos, 1991) is used to map the space of stochastic system states conditioned on a realization of the white noise disturbance processes to the space of coefficients of the polynomial chaos expansions of states. The effect of white noise processes on system dynamics is then efficiently accounted for in the space of expansions' coefficients assuming that the coefficients possess a Gaussian distribution. To update the uncertainty description of parameters based on measurements, a gPC-based histogram filter (Bavdekar and Mesbah, 2016) is utilized to estimate the probability distributions of uncertain parameters at each measurement sampling time. The updated probability distributions of parameters are used to adapt the polynomial basis functions in the controller. A sample-based approach is adopted to approximate the joint chance constraints (Alamo et al., 2010). The performance of the proposed SNMPC approach is demonstrated for regulating the thermal effects of an atmospheric-pressure plasma jet that is developed for biomedical applications (Gidon et al., 2016).

Notation. $\mathbb{R}$ and $\mathbb{N} = \{1, 2, \ldots\}$ denote the set of real numbers and natural numbers, respectively. $(\Omega, \mathcal{F}, \mathsf{P})$ denotes the probability space with Ω, $\mathcal{F}$, and P being the sample space, σ-algebra, and probability distribution function (pdf) on Ω, respectively. $\mathsf{P}(\cdot|z)$ denotes the pdf of a stochastic variable conditioned on z. $\mathcal{N}(z; \mu, \Sigma)$ denotes that a stochastic variable z has a Gaussian distribution with mean μ and covariance Σ. $\mathbf{Pr}[\cdot]$ denotes probability. $\mathbf{E}[\cdot]$ and $\mathbf{Var}[\cdot]$ denote the expected value and variance of a stochastic variable, respectively.

2. PROBLEM FORMULATION

Consider a discrete-time nonlinear system

$$x(t) = f\big(x(t-1), \theta, u(t-1)\big) + w(t-1), \qquad (1)$$

where t denotes the time index; $x(t) \in \mathbb{R}^n$ denotes the system states with uncertain initial conditions $x(0) \sim \mathsf{P}_{x_0}$; $u(t) \in \mathbb{R}^m$ denotes the system inputs; $\theta \in \mathbb{R}^p$ denotes the uncertain system parameters characterized by the pdf P_θ; $w(\omega(t)) \sim \mathcal{N}(\omega; 0, Q)$ denotes zero-mean white noise processes with the known covariance matrix $Q \in \mathbb{R}^{n\times n}$; and $f : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^p \to \mathbb{R}^n$ denotes the vector function of nonlinear system dynamics. The probabilistic uncertainties $[x_0^\top \ \theta^\top]^\top \in \mathbb{R}^{n_\xi}$, $(n_\xi \leq n+p)$ are defined in terms of the standard random variables $\xi \in \mathbb{R}^{n_\xi}$, which belong to the Hilbert space $\mathcal{L}^2\big(\Omega, \mathcal{F}, \mathsf{P}\big)$. The standard random variables $\{\xi_j\}_{j=1}^{n_\xi}$ are independently distributed with arbitrary, but known pdfs P_{ξ_j} that have a finite variance. The measurements of states $x(t)$ are corrupted by zero-mean Gaussian noise with covariance $R \in \mathbb{R}^{n\times n}$.

Assumption 1. In (1), f consists of polynomial functions in x and θ.[1]

The inputs in (1) are subject to hard constraints

$$u(t) \in \mathbb{U} := \{u(t) \in \mathbb{R}^m \,|\, H_u u(t) \leq d_u\}, \qquad (2)$$

where $H_u \in \mathbb{R}^{s\times m}$; $d_u \in \mathbb{R}^s$; and $s \in \mathbb{N}$ denotes the number of input constraints. The stochastic system states must satisfy a joint chance constraint (JCC) of the form

$$\mathbf{Pr}\left[g_i(x(t)) \leq 0, \quad \forall i = 1, \ldots, l\right] \geq \beta, \qquad (3)$$

where $g_i : \mathbb{R}^n \to \mathbb{R}$ denotes a possibly nonlinear vector function of state constraints; $l \in \mathbb{N}$ denotes the number of state constraints; and $\beta \in (0, 1]$ is the lower bound for the probability level that the JCC must be satisfied. The JCC ensures that the state constraints are fulfilled with an admissible probability level in the presence of the stochastic uncertainties in (1).

Under the assumption of full-state feedback, this paper presents a SNMPC approach for the stochastic nonlinear system (1) subject to the hard input constraints (2) and the JCC (3). SNMPC involves solving a stochastic optimal control problem (OCP) in a receding-horizon manner. The main challenge in solving the stochastic OCP lies in efficiently predicting the time-evolution of the probability distributions (or statistics) of the stochastic states. This work adopts the gPC framework (Xiu and Karniadakis, 2002) for uncertainty propagation. The gPC-based SNMPC approach in (Mesbah et al., 2014) is extended to enable propagating the system disturbances $w(t)$ (along with $[x_0^\top \ \theta^\top]^\top$) as well as handling of the JCC (3). A gPC-based Bayesian estimator (Bavdekar and Mesbah, 2016) is solved in tandem with the stochastic OCP to estimate the probability distributions of the uncertain parameters θ at each sampling time. The estimated pdfs of parameters and the measured pdf of states are used to facilitate receding-horizon implementation of the SNMPC approach. The uncertainty propagation method used to arrive at a computationally tractable formulation for the stochastic OCP is introduced in the next section.

[1] Nonpolynomial functions can be converted to a polynomial form if f is analytic and separable with respect to x and θ (Papachristodoulou and Prajna, 2005).