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Spatial Formation Control with Volume Information: Application to Quadcopter UAV's ⋆

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Abstract This paper extends the distance-based formation control for the case of holonomic robots moving in 3D space. The approach is addressed for agents modeled as double-integrators with any undirected communication graphs. The control strategy uses a combined distance-based attractive-repulsive potentials to ensure convergence to the formation pattern avoiding possible inter-robot collisions. In order to avoid unwanted formation patterns that verify the distance constraints, each robot control law includes a volume condition which provides information about the unique desired position of each robot in the formation pattern. The proposed algorithm is tested by numerical simulations and extended to the case of quadcopters UAV's by an input-output linearization showing good behavior.

Keywords: Mobile robots; Formation control; Artificial Potential Function; Collision Avoidance; Quadcopter UAV.

1. INTRODUCTION

Distance-based formation control (DFC) is a fundamental issue in the motion coordination of multi-robot systems (Oh et al., 2015). The main challenge is to design decentralized control strategies to move the robots to a desired formation pattern defined by distance constraints, avoiding inter-robot collisions. The control strategies encompass from reactive algorithms based in natural-behaviors (Yu and LaValle, 2012) to the use of communication graphs and distance-based functions with attractive and repulsive behavior in Dimarogonas and Johansson (2008).

The main drawback of the DFC with respect to the traditional position-based formation control widely studied in (Ren and Beard, 2008) and (Hernandez-Martinez and Aranda-Bricaire, 2011) falls in the generation of different final configurations of robots that satisfy the distance constraints.Therefore, rigidity problems arise (Krick et al., 2009). In this sense, a possible solution to achieve an unique formation pattern is to construct rigid patterns where at least $(n-3)$ communication edges for n robots must be defined as shown in (Olfati-Saber and Murray, 2002; Krick et al., 2008). Other approaches use additional information in the formation setup, such as absolute angle respect to a leader (Desai et al., 2001) or internal angles as studied in a previous work in (Ferreira-Vazquez et al., 2015).

Most of the DFC approaches have been addressed for the case of single-integrators robots moving in the plane, like in Dimarogonas and Johansson (2008) for the case of tree-shaped formations, DFC with cycles in (Dimarogonas and Johansson, 2009) or specific leader-followers schemes focus in the application to wheeled mobile robots in (Desai et al., 2001; Toibero et al., 2008). Feedback linearization techniques have been used when robot kinematics are more complex in (Liu and Jiang, 2013). The convergence of formations with mismatched distance constraints between agents is addressed in (Helmke et al., 2014). The DFC applied to the case of double-integrators is studied in Dimarogonas and Johansson (2008); Anderson et al. (2012) for the leader-followers scheme and Oh and Ahn (2014) for the case of undirected communication graphs using a gradient-like control law.

The extension to the case of agents moving in 3D space allows the application to formations of UAV's. Position-based formation control of quadcopters is studied in Alfriend et al. (2010); Sumano et al. (2013) using a reduce model of the position dynamics. Sliding mode control combined with neural networks is proposed in Bo and Gao (2009). The leader-follower scheme of distance and relative angle in UAV's is proposed in (Guangyan and Zheng, 2014) where an elastic term is added providing a better control effect. Collision avoidance of formation using Model Predictive Control (MPC) in discrete-time is given in (Chao et al., 2011), where the robots are formed with respect to a reference point. Finally, in

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(Nielsen and Sharma, 2015) a DFC scheme is designed using the Lyapunov method, where the each robot keeps the leader vehicle at a desired bearing angle in its field-of-view using a video camera. In all the previous works, it is supposed an inner control related to the orientation angles of UAV's and the reduction of the position dynamics as a unicycle-type robot moving in a plane.

In this paper, we extend our previous work in (Ferreira-Vazquez et al., 2015) for the case of $3D$ for spacial DFC and double integrator models, different to the absolute position-based formation in (Alfriend et al., 2010; Sumano et al., 2013). The formation strategy is based on artificial potential functions with attractive-repulsive behavior combined with information about the position of each robot respect to other three robots using a triple product. To the best of our knowledge, the triple product artificial potential has not been used in other works. The previous condition ensures the convergence to a unique formation pattern. The approach is applied to the case of quadcopter UAV's formations using input-output feedback linearization previously proposed in (Hernandez-Martinez et al., 2015), avoiding a model reduction of the orientational or translational dynamics as presented in (Guangyan and Zheng, 2014). Thus, the control approach becomes decentralized where the UAV's require to sense the displacement coordinates respect to its adjacent members defined by any well-defined and rigid undirected communication graph, becoming a more general result than the leader-follower setups given in (Bo and Gao, 2009; Chao et al., 2011). Simulations for the case of four UAV's with a virtual reality environment show the feasibility of the approach.

The paper is organized as follows. Section 2 formulates the formation problem. In Section 3, the control strategy is presented and the convergence is analyzed by Lyapunov techniques. Section 4 extends the result to the case of quadcopter UAV's with numerical simulations. Finally, section 5 presents some conclusion remarks and future work.

2. PROBLEM DEFINITION

2.1 Modified distance-based Formation Problem

Let $\mathbf{q} = [\mathbf{q}_1, \ldots, \mathbf{q}_N]^\top \in \mathbb{R}^{3\times N}$ be the position of the center of N three-dimensional omnidirectional robots of diameter ρ_i with the double integrator dynamics given by the following equation:

$$\begin{aligned} \dot{\mathbf{q}}_i &= \mathbf{p}_i, \\ \dot{\mathbf{p}}_i &= \mathbf{u}_i, \end{aligned} \quad i = 1, \ldots, N \tag{1}$$

Let $\mathbf{r}_{ij} = \mathbf{q}_j - \mathbf{q}_i$ be the relative position vector of robot R_j with respect to robot R_i and

$$r_{ij} = \|\mathbf{r}_{ij}\| \tag{2}$$

be the Euclidean distance between robots R_i and R_j. A distance topology is defined by the sets $N_i, i = 1, \ldots, N$ of robot indexes j for which a desired distance d_{ij} between robots R_j and R_i is defined. It is assumed the topology is bidirectional, i.e. that $j \in N_i \implies i \in N_j$.

For any 4-tuple (i, j, k, m) the 3-simplex oriented volume define by robots R_i, R_j, R_k and R_m (Boyd and Vandenberghe, 2004) can be expressed by

$$\alpha_{ijkm} = \frac{1}{6}\mathbf{r}_{ij}^\top(\mathbf{r}_{ik} \times \mathbf{r}_{im}). \tag{3}$$

Figure 1 shows a representation of α_{ijkm} in space. This volume provides relative angular information between robots

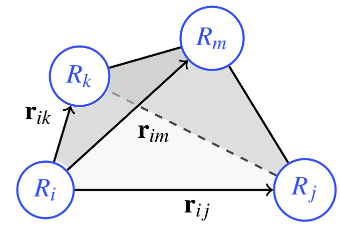

Figure 1. The value of α_{ijkm} is proportional to the tetrahedron volume enclosed by the robots R_i, R_j, R_k and R_m positions.

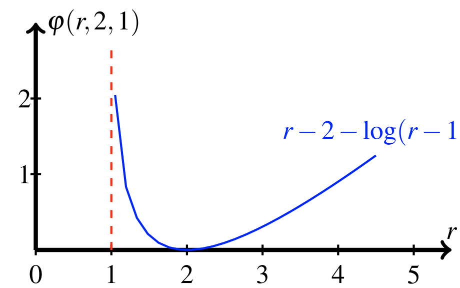

Figure 2. Combined attractive-repulsive artificial potential field as a function of r for the case where $d_{ij} = 2$ and $c_{ij} = 1$.

in a configuration $\mathbf{q}$. Let M be the set of all 4-tuples $(i_s, j_s, k_s, m_s), s = 1, \ldots, L$, for which a desired simplex volume $\alpha^\star_{i_s j_s k_s m_s}$ is defined. Taking into account the set M, a more restricted distance-based problem may be formulated as the following:

Problem Statement 1. Given N robots with dynamics defined by (1), find a control law for $\mathbf{u}_i, \forall i$ such that the robots converge without collisions to a configuration $\mathbf{q}$ verifying distance constraints $r_{ij} = d_{ij}$, $\forall j \in N_i$ and volume constraints $\alpha_{ijkm} = \alpha^\star_{ijkm}$, $\forall (i, j, k, m) \in M$.

A formulation to solve this problem is presented next.

3. FORMATION CONTROL

3.1 Combined attractive-repulsive artificial potential field

In order to converge to pre-specified inter-robot distances without collisions, an attractive-repulsive artificial potential field is used, given by the following definition.

Definition 1. Given any two robots R_i and R_j an Artificial Potential Field φ_{ij} is defined as:

$$\varphi_{ij}(r_{ij}, d_{ij}, c_{ij}) = \frac{r_{ij} - d_{ij}}{d_{ij} - c_{ij}} - \log\left(\frac{r_{ij} - c_{ij}}{d_{ij} - c_{ij}}\right) \tag{4}$$

with r_{ij} given by (2), d_{ij} their desired distance and

$$c_{ij} = \frac{1}{2}(\rho_i + \rho_j) \tag{5}$$

their collision distance where ρ_k, $k = 1, \ldots, N$ the robot diameters and $d_{ij} > c_{ij}$.

Figure 2 shows the shape of the combined potential field. Observe also that φ_{ij} is well defined for $r_{ij} > c_{ij}$ and tends to $+\infty$ when $r_{ij} \to c_{ij}$ with $r_{ij} > c_{ij}$.

3.2 Repulsive potential field

In a general formation specification there will be robots that do not communicate with each other due to several reasons. For