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A method for adaptive hybrid output regulation of linear systems

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Abstract: In this paper we deal with the problem of *adaptive* output regulation for a class of uncertain hybrid linear systems whose state jumps periodically according to a known clock. The goal is to present a design methodology for internal model-based hybrid regulators in presence of hybrid exosystems that are *unknown* both in the flow dynamics and jump map, which are assumed to range in a finite set. The arguments used in this paper strongly rely on the ones recently presented in Cox et al. [2016] and, in particular, on the notion of visibility of the "hybrid steady-state generator". The design methodology is then applied to the practical case of torque control of a DC-motor in presence of an uncontrolled rectifier.

Keywords: Hybrid systems, regulation, internal model, observability.

1. INTRODUCTION

The problem of output regulation has attracted extensive attention in the literature since the milestone paper Francis & Wonham [1976] in which the internal model principle has been introduced for the robust regulator design of linear systems. The problem has played a central role also in the literature of nonlinear control systems starting with the fundamental paper Isidori & Byrnes [1990] in which the theoretical base for the nonlinear internal model-based regulator has been laid. Starting from that work, much progress has been made (Isidori [1997], Marconi & Praly [2008], Marino & Tomei [2011], to mention a few), aiming at systematically designing a (robust) internal model-based regulator for continuous-time systems. Within this framework a special mention goes to the work Serrani et al. [2001] in which a regulator design in presence of uncertainties on the exosystem dynamics has been proposed, by thus solving a problem of *adaptive* output regulation.

While the theory of robust output regulation for continuous-time systems has arrived at a mature stage, the extension of the theory to hybrid systems is still at an early stage. In this respect, the work Marconi & Teel [2013] has laid the foundations for the development of a theory of output regulation for a specific class of linear hybrid systems, in which the state of a continuous-time system periodically jumps according to the value of a clock variable assumed to be known in the regulator design. In that work key notions, such as the notion of steady state, of regulator equations and internal model property, well-known for continuous-time systems, have been generalised to that hybrid setting. Starting from the framework in Marconi & Teel [2013], a number of results has been developed subsequently. Cox et al. [2011] investigated the asymptotic regulation problem for the system whose clock variable is not measured by means of a hybrid clock phase estimator. In Carnevale et al. [2013], the necessary and sufficient conditions for output regulation were proposed for both full information and error feedback case under the assumption that the hybrid regulator is time-invariant. In addition to these hybrid linear results, there are also many interesting results related to nonlinear hybrid output regulation, such as Cox et al. [2013] and Forte et al. [2014]. Within these research attempts, it is worth mentioning the recent contribution in Cox et al. [2016] in which a systematic design approach for designing *robust* internal model-based hybrid regulators is presented, by overtaking some intrinsic limitations of the design methods proposed in Marconi & Teel [2013] in terms of robustness. The method proposed in that paper takes advantage of the notion of "invisibility" originally proposed in D'Alessandro et al. [1973] in the study of observability for time-varying linear systems, and relies on the idea of isolating "invisible dynamics" from the so-called "hybrid stead-state generator".

In this paper, we aim at moving preliminary steps towards the design of hybrid internal-model based regulators in the framework of Marconi & Teel [2013] by considering the case in which the hybrid exosystem dynamics, both in the flow and jump part, are unknown. The result can be framed in a context of *adaptive* hybrid output regulation in the sense that we aim to extend the result in Serrani et al. [2001] to a specific hybrid framework. In the paper we consider the case in which the flow and jump exosystem dynamics, although unknown, belong to a known finite set. The proposed solution borrows the framework proposed in Cox et al. [2016] and constructs the regulator by isolating "invisible" dynamics from "hybrid steady-state generator" obtained by making a replica of all possible exosystem dynamics that might occur due to the uncertainties.

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The paper is laid out as follows. Section 2 presents the preliminaries and some known concepts that are then used in the regulator design. In Section 3, by applying the notion of state-output equivalence of "hybrid steady-state generators" we design the robust regulator. In Section 4, the proposed methodology is applied to the practical example of torque control of a DC-motor in presence of an uncontrolled rectifier, by showing the effectiveness of the proposed design methodology by means of simulations. Finally, Section 5 conclude the paper by presenting possible future developments.

2. THE FRAMEWORK AND PRELIMINARY RESULTS

2.1 The framework

In this paper, we consider a SISO hybrid linear system flowing according to

$$\begin{aligned} \dot{\tau} &= 1 \\ \dot{z} &= A_{11}(\theta)z + A_{12}(\theta)x_1 + P_1(\theta)w \\ \dot{x}_i &= x_{i+1} + P_{2,i}(\theta)w\,, \qquad 1 \le i \le r-1 \\ \dot{x}_r &= A_{31}(\theta)z + A_{32}(\theta)x + b(\theta)u + P_{2,r}(\theta)w \end{aligned} \tag{1}$$

when $(\tau, z, x) \in [0, \tau_{max}] \times \mathbb{R}^{(n-r)} \times \mathbb{R}^r$, and jumping according to

$$\begin{aligned} \tau^+ &= 0 \\ z^+ &= M_{11}(\theta)z + M_{12}(\theta)x + N_1(\theta)w \\ x^+ &= M_{21}(\theta)z + M_{22}(\theta)x + N_2(\theta)w \end{aligned} \tag{2}$$

when $(\tau, z, x) \in \{\tau_{max}\} \times \mathbb{R}^{(n-r)} \times \mathbb{R}^r$, in which $(z, x) \in \mathbb{R}^{(n-r)} \times \mathbb{R}^r$, with $x = \text{col}\,(x_1, x_2, \cdots, x_r) \in \mathbb{R}^r$, $u \in \mathbb{R}$ and $x_1 \in \mathbb{R}$ are, respectively, the state, the control input and the measurable output, w is an exogenous variable, and $\theta \in \Theta$ is a vector of constant uncertainties with $\Theta \in \mathbb{R}^{n_0}$ a fixed compact set. Flow and jump times of the previous system are governed by the value of the variable τ playing the role of clock variable with τ_{max} denoting a dwell-time between two consecutive state jumps. In the flow dynamics of the previous system $r \ge 1$ represents the relative degree of the flow dynamics between the input u and the output x_1. The hybrid framework considered here is consistent with the one presented in Goebel et al. [2012].

Consistently with the literature on hybrid output regulation, w is supposed to be generated by a hybrid exosystem modelled by

$$\left.\begin{aligned} \dot{\tau} &= 1 \\ \dot{w} &= Sw \end{aligned}\right\} (\tau, w) \in [0, \tau_{max}] \times \mathbb{R}^s \qquad \left.\begin{aligned} \tau^+ &= 0 \\ w^+ &= Jw \end{aligned}\right\} (\tau, w) \in \{\tau_{max}\} \times \mathbb{R}^s \tag{3}$$

where $w \in W \subset \mathbb{R}^s$ with W a compact set and $[0, \tau_{\max}] \times W$ forward invariant for (3). Associated to the previous system, the regulated error is defined by

$$e = x_1 + Qw$$

and the final goal is to design an error feedback (hybrid) regulator able to steer asymptotically the regulation error to zero (as the sum of ordinary time and the number of jumps tends to infinity) for all possible initial conditions $(w(0,0), (z(0,0), x(0,0))) \in W \times \mathbb{R}^n$.

The main goal of the paper is to deal with the previous problem in the case in which the exosystem is uncertain both in S and J. The simplified framework that is considered here is to assume that S and J can take values in finite known sets, namely

$$\begin{aligned} S &\in \mathcal{S} := \{S_1, S_2, \cdots, S_{n_1}\} \\ J &\in \mathcal{J} := \{J_1, J_2, \cdots, J_{n_2}\}\,. \end{aligned}$$

with S_j, $j = 1, \ldots, n_1$, and J_j, $j = 1, \ldots, n_2$, known matrices.

Furthermore, in order to simplify the design of the stabiliser in the forthcoming developments, we assume (as in Cox et al. [2016]) that system (1) is minimum-phase as detailed next.

Assumption 1. The pair $(A_{11}(\theta), M_{11}(\theta))$ is such that the eigenvalues of the matrix $M_{11}(\theta)\exp(A_{11}(\theta)\tau_{max})$ are within the unitary disk for all $\theta \in \Theta$.

2.2 Preliminaries

According to the results in Marconi & Teel [2013], a necessary condition for the existence of the hybrid regulator solving the problem at hand is that there exist continuous functions $\Pi : [0, \tau_{\max}] \to \mathbb{R}^{n \times s}$ and $R : [0, \tau_{\max}] \to \mathbb{R}^{1 \times s}$ such that

$$\begin{aligned} \frac{d\Pi}{d\tau}(\tau) &= A\Pi(\tau) - \Pi(\tau)S + P + BR(\tau) \\ \Pi(0)J &= M\Pi(\tau_{\max}) + N \\ 0 &= C\Pi(\tau) + Q\,, \end{aligned} \tag{4}$$

where $\Pi(\tau)$ is partitioned as col $(\Pi_z(\tau), \Pi_x(\tau))$ coherently with the state partition col (z, x), and

$$A = \begin{pmatrix} A_{11} & A_{12}C \\ 0 & A_{22} \\ A_{31} & A_{32} \end{pmatrix}, \quad P = \begin{pmatrix} P_1 \\ P_{2,1} \\ \cdots \\ P_{2,r} \end{pmatrix},$$

$$M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}, \quad N = \begin{pmatrix} N_1 \\ N_2 \end{pmatrix},$$

$B = \text{col}\,(0, \cdots, 0, b) \in \mathbb{R}^n$, $C \in \mathbb{R}^{1 \times n}$ is a row vector whose components are zeros except the $n-r+1$-th element which is one and $A_{22} = (\,0 \quad I_{r-1}\,) \in \mathbb{R}^{(r-1) \times r}$. It is worth noting that the solutions $(\Pi(\tau), R(\tau))$ are, in general, uncertain since they depend on θ and, in our "adaptive" scenario, on S and J.

We thus continue the analysis by assuming the existence of $(\Pi(\tau), R(\tau))$ by linking the reader to Cox et al. [2016] for sufficient conditions (involving the structure of the matrix M) under which such a solution exists. A crucial role in the regulator construction is played by the so-called *hybrid steady-state generator system*, which is the system with output y_w described by

$$\begin{aligned} &\left.\begin{aligned} \dot{\tau} &= 1 \\ \dot{w} &= Sw \end{aligned}\right\} \quad (\tau, w) \in [0, \tau_{max}] \times W \\ &\left.\begin{aligned} \tau^+ &= 0 \\ w^+ &= Jw \end{aligned}\right\} \quad (\tau, w) \in \{\tau_{max}\} \times W \\ &y_w = R(\tau)w\,. \end{aligned} \tag{5}$$