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Self-triggered control via dynamic high-gain scaling

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Abstract: This paper focuses on the construction of self-triggered state feedback laws. The approach followed is a high-gain approach. The event which triggers an update of the control law is based on an dynamical system in which the state is the high-gain parameter. This approach allows to design a control law ensuring convergence to the origin for nonlinear systems with triangular structure and a specific upper bound on the nonlinearities.

Keywords: Nonlinear systems, self-triggered control, continuous-discrete time controller, high-gain, dynamic scaling.

1. INTRODUCTION

The implementation of a control law on a process requires the use of an appropriate sampling scheme. In this regards, periodic control (with a constant sampling period) is the usual approach that is followed for practical implementation on digital platforms. Indeed, periodic control benefits from a huge literature, providing a mature theoretical background (see e.g. Aström and Wittenmark (1997); Nesic et al. (1999); Mazenc et al. (2015); Dinh et al. (2015)) and numerous practical examples. The use of a constant sampling period makes easier the closed-loop analysis and the implementation, allowing solid theoretical results and a wide deployment in the industry. However, the rate of control execution being fixed by a worst case analysis (the chosen period must guarantee the stability for all possible operating conditions), this may lead to an unnecessary fast sampling rate and then to an overconsumption of available resources.

The recent growth of shared networked control systems for which communication and energy resources are often limited goes with an increasing interest in aperiodic control design. This can be observed in the comprehensive overview on event-triggered and self-triggered control presented in Heemels et al. (2014). Event-triggered control strategies introduce a triggering condition assuming a continuous monitoring of the plant (that requires a dedicated hardware) while in self-triggered strategies, the control update time is based on predictions using previously received data. The main drawback of self-triggered control is the difficulty to guarantee an acceptable degree of robustness, especially in the case of uncertain systems.

Most of the existing results on event-triggered and self-triggered control for nonlinear systems are based on the input-to-state stability (ISS) assumption which implies the existence of a feedback control law ensuring an ISS property with respect to measurement errors (Tabuada (2007); Anta and Tabuada (2010); Abdelrahim et al. (2015); Postoyan et al. (2015)).
In this ISS framework, an emulation approach is followed: the knowledge of an existing robust feedback law in continuous time is assumed then some triggering conditions are proposed to preserve stability under sampling (see also the approach of Seuret et al. (2013)).

Another proposed approach consists in the redesign of a continuous time stabilizing control. For instance, the authors of Marchand et al. (2013) adapted the original *universal formula* introduced by Sontag for nonlinear systems affine in the control. The relevance of this method was experimentally shown in Villarreal-Cervantes et al. (2015) where the regulation of an omnidirectional mobile robot was addressed.

Although aperiodic control literature has proved an interesting potential, important fields still need to be further investigated to allow a wider practical deployment.

The high-gain approach is a very efficient tool to address the stabilizing control problem in the continuous time case. It has the advantage to allow uncertainties in the model and to remain simple.

Different approaches based on high-gain techniques have been followed in the literature to tackle the output feedback problem in the continuous-time case (see for instance Andrieu et al. (2009a), Krishnamurthy and Khorrami (2004), Andrieu and Tarbouriech (2013)) and more recently for the (periodic) discrete-in-time case (see Qian

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and Du (2012)). In the context of observer design, Andrieu et al. (2015) proposed the design of a continuous discrete time observer, revisiting high-gain techniques in order to give an adaptive sampling stepsize.

In this work, we extend the results obtained in Andrieu et al. (2015) to self-triggered state feedback control. In high-gain designs, the asymptotic convergence is obtained by dominating the nonlinearities with high-gain techniques. In the proposed approach, the high-gain is dynamically adapted with respect to time varying nonlinearities in order to allow an efficient trade-off between the high-gain parameter and the sampling step size. Moreover, the proposed strategy is shown to ensure the existence of a minimum inter-execution time.

The paper is organized as follows. The control problem and the class of considered systems is given in Section 2. In Section 3, some preliminary results concerning linear systems are given. The main result is stated in Section 4 and its proof is given in Section 5. Finally Section 6 contains an illustrative example.

2. PROBLEM STATEMENT

2.1 Class of considered systems

In this work, we consider the problem of designing a self-triggered control for the class of uncertain nonlinear systems described by the dynamical system

$$\dot{x}(t) = Ax(t) + Bu(t) + f(x(t)), \tag{1}$$

where the state x is in $\mathbb{R}^n$, $u : \mathbb{R} \to \mathbb{R}$ is the control signal in $\mathbb{L}^\infty(\mathbb{R}_+, \mathbb{R})$, A is a matrix in $\mathbb{R}^{n\times n}$, B is a vector in $\mathbb{R}^{n\times 1}$ and $f : \mathbb{R}^n \to \mathbb{R}^n$ is a vector field having the following triangular structure

$$A = \begin{pmatrix} 0 & 1 & & (0) \\ & \ddots & \ddots & \\ & & 0 & 1 \\ (0) & & & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}, \tag{2}$$

$$f(x) = \begin{pmatrix} f_1(x_1) \\ f_2(x_1, x_2) \\ \vdots \\ f_n(x_1, x_2, \ldots, x_n) \end{pmatrix}. \tag{3}$$

We consider the case in which the vector field f satisfies the following assumption.

Assumption 2.1. (Nonlinear bound). There exist a nonnegative continuous function c, positive real numbers c_0, c_1 and q such that for all $x \in \mathbb{R}^n$, we have

$$|f_j(x(t))| \leqslant c(x_1)\left(|x_1| + |x_2| + \cdots + |x_j|\right), \tag{4}$$

with

$$c(x_1) = c_0 + c_1|x_1|^q. \tag{5}$$

Notice that Assumption 2.1 is more general than the incremental property introduced in Qian and Du (2012) since the function c is not constant but depends on x_1. This bound can be related also to Praly (2003); Krishnamurthy and Khorrami (2004) in which continuous output feedback law are designed. However, in these works no bounds are imposed on the function c. Note moreover that in our context we don't consider inverse dynamics.

2.2 Updated sampling time controller

The design of a self-triggered controller involves to compute the sequence of control values $u(t_k)$ where $(t_k)_{k\geqslant 0}$ is a sequence of times to be selected. We refer to the instants t_k as *execution times*. The existence of a *minimal inter-execution time*, which is some bound $\delta > 0$ such that $t_{k+1} - t_k \geqslant \delta$ for all $k \geqslant 0$, is needed to avoid zero inter-sampling time leading to Zeno phenomena.

In the sequel, we restrict ourselves to a classic sample-and-hold implementation, i.e., the input is constant between any two execution times: $u(t) = u(t_k), \quad \forall t \in [t_k, t_{k+1})$. Hence, in addition to a feedback controller that computes the control input, event-triggered and self-triggered control systems need a *triggering mechanism* that determines when the control input has to be updated again. This rule is said to be *static* if it only involves the current state of the system, and *dynamic* if it uses an additional internal dynamic variable (see Girard (2015)).

2.3 Notation

We denote by $\langle\cdot,\cdot\rangle$ the canonical scalar product on $\mathbb{R}^n$ and by $\|\cdot\|$ the induced Euclidean norm; we use the same notation for the corresponding induced matrix norm. Also, we use the symbol $'$ to denote the transposition operation.

In the following, the notation $\xi(t^-)$ stands for $\lim_{\substack{\tau\to t\\ \tau<t}} \xi(\tau)$. Also, to simplify the presentation, we introduce the notations $\xi_k = \xi(t_k)$ and $\xi_k^- = \xi(t_k^-)$.

3. PRELIMINARY RESULTS: THE LINEAR CASE

In high-gain designs, the idea is to consider the nonlinear terms (the f_i's) as disturbances. A first step consists in synthesizing a robust control for the linear part of the system, neglecting the effects of the nonlinearities. Then, the convergence and robustness are amplified through a high gain parameter to deal with the nonlinearities.

Therefore, let us first focus on a general linear dynamical system

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{6}$$

where the state x evolves in $\mathbb{R}^n$ and the control u is in $\mathbb{R}$. The matrix A is in $\mathbb{R}^{n\times n}$ and B is a column vector in $\mathbb{R}^n$.

In this preliminary case, we review a well-known result concerning periodic sampling approaches. Indeed, an emulation approach is adopted for the stabilization of the linear part: a feedback law is designed in continuous time and a triggering condition is chosen to preserve stability under sampling.

It is well known that if there exists a feedback control law (continuous-in-time) $u(t) = Kx(t)$ that asymptotically stabilizes the system then there exists a strictly positive inter-execution time $\delta_k = t_{k+1} - t_k$ such that the discrete-in-time control law $u(t) = Kx(t_k)$ for t in $[t_k, t_{k+1})$ renders the system asymptotically stable. This result is rephrased in Lemma 3.1 below whose proof can be found in Peralez et al. (2015) and for which we do not claim any originality.

Lemma 3.1. Suppose the pair (A, B) is stabilizable, that is there exists a matrix K in $\mathbb{R}^n$ rendering $(A+BK)$ Hurwitz.