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Swing up and stabilization of the Acrobot via nonlinear optimal control based on stable manifold method

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Abstract: This paper considers the problem of swing up and stabilization for the Acrobot. It is shown that stable manifold method which has been proposed for computing nonlinear optimal control is capable of designing feedback controllers for this problem. An optimal stabilization controller is obtained as a single feedback law by numerically solving a Hamilton-Jacobi equation by the stable manifold method. It is shown that unlike existing methods for Acrobot swing up such as partial feedback linearization, the resultant control is mechanically indigenous in the sense that it uses reactions of arms effectively and, as a consequence, control input is kept low. A number of simulations verify the effectiveness and robustness of the controller.

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1. INTRODUCTION

Underactuated systems are mechanical systems which have fewer control inputs than degrees of freedom. Control of underactuated systems is currently an active topic for many researchers due to wide application range in Robotics or aerospace field (Liu and Yu, 2013; Xin and Liu, 2013). The Acrobot is a 2-dimensional underactuated mechanical system often used as a benchmark problem for testing nonlinear control methods. It consists of two links and an actuator is installed only at the second joint. A common control objective of the Acrobot is to swing it up from the downward position to the unstable upright position and to stabilize it vertically. This is a challenging task because of the movement in a large range of nonlinearity.

Generally, the swing up control is divided into two phases, first, the swing up phase in which nonlinearity is dominant, and then, stabilization phase which estabishes autonomus stability in a neighborhood of the origin. There are some other effective ways to design swing up controllers such as using partial feedback linearization (Spong, 1995), energy feedback (Xin and Yamasaki, 2012; Xin and Kaneda, 2007), trajectory tracking (Zhang et al., 2013), Lyapunov based control (Zergeroglu et al., 1998) and intelligent control (Brown and Passino, 1997). However, switching controller has no guarantee of stability in the vicinity of the boundary. Researchers in (Davison and Bortoff, 1997; Xin and Kaneda, 2001) propose methods to enlarge the region of attraction (RoA) of linear controllers, stabilization of the Acrobot by linear control is inherently difficult. Backstepping approachOlfati-Saber (2000) should be mentioned as a single feedback control method under some assumptions that are difficult to satisfy generally.

In this paper, we show that it is possible to design a single (without switching) optimal feedback controller for swing up and stabilization of the Acrobot using the stable manifold method (Sakamoto and van der Schaft, 2008; Sakamoto, 2013). The method has been developed for numerically computing the derivative of solution for Hamilton-Jacobi equations (HJEs). When it is applied for the Acrobot swing up problem, it directly enlarges the RoA for stabilization so that the downward position is included in RoA. For a survey and other solution methods for HJEs, we refer to Aguilar and Krener (2014); Aliyu (2011); Beeler et al. (2000); Lukes (1969); Navasca and Krener (2007).

The organization of the paper is as follows. The Acrobot model is introduced in § 2. § 3 summarizes the theory of the stable manifold method for HJEs. Controller design is precisely explained in § 5 and simulation results are shown in § 6.

2. MODELING AND ANALYSIS OF THE ACROBOT

In this section, we derive a nonlinear model of the Acrobot. Figs. 1, 2 show the Acrobot and its schematic model. The control torque is applied only to the second joint from an actuator through a pulley and a timing belt.

For the ith link ($i = 1, 2$), q_i is the angle, m_i is the mass, l_i is the length, l_{ci} is the distance from ith joint to the center of mass (COM), J_i is the inertia around the center of mass, and let g be a gravitational acceleration (9.801

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m/s^2). The equation of motion of the Acrobot is derived as follows by the method of Lagrange.

$$M(q_2)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau, \quad (1)$$

where

$$M(q_2) = \begin{bmatrix} M_{11}(q_2) & M_{12}(q_2) \\ M_{21}(q_2) & M_{22} \end{bmatrix} = \begin{bmatrix} a_1 + a_2 + 2a_3\cos q_2 & a_2 + a_3\cos q_2 \\ a_2 + a_3\cos q_2 & a_2 \end{bmatrix}$$

$$C(q,\dot{q}) = \begin{bmatrix} -a_3\dot{q}_2\sin q_2 & -a_3(\dot{q}_1+\dot{q}_2)\sin q_2 \\ a_3\dot{q}_1\sin q_2 & 0 \end{bmatrix},$$

$$G(q) = \begin{bmatrix} -b_1\sin q_1 - b_2\sin(q_1+q_2) \\ -b_2\sin(q_1+q_2) \end{bmatrix},$$

$$a_1 = m_1 L_{c1}^2 + m_2 L_1^2 + J_1, \quad a_2 = m_2 L_{c2}^2 + J_2,$$
$$a_3 = m_2 L_1 L_{c2}, \quad b_1 = (m_1 L_{c1} + m_2 L_1)g,$$
$$b_2 = m_2 L_{c2} g.$$

$\tau = [\tau_1,\ \tau_2]^T$ are the torque on the joints. We assume that there exists resistance force proportional with angular velocity such as viscous friction or counter electromotive force of the actuator. Then τ is given as

$$\tau_1 = -\mu_1\dot{q}_1,$$
$$\tau_2 = nK_{DC}u - \mu_2\dot{q}_2,$$

where u is the control input voltage for the actuator, n is gear ratio of the pulley, K_{DC} is electromotive torque constant and μ_1, μ_2 is the viscous resistance coefficient of the joints.

Defining $x = [x_1, x_2, x_3, x_4]^T = [q_1, q_2, \dot{q}_1, \dot{q}_2]^T$ as system variables, and letting $H(q,\dot{q}) = [H_1(q,\dot{q}), H_2(q,\dot{q})]^T = C(q,\dot{q})\dot{q} + G(q) + [\mu_1\dot{q}_1,\ \mu_2\dot{q}_2]^T$, the dynamic equation (1) is rewritten in the state space form as

$$\dot{x} = f(x) + g(x)u, \quad (2)$$

where

$$f(x) = \begin{bmatrix} x_3 \\ x_4 \\ -M^{-1}(q_2)\begin{bmatrix} H_1(q,\dot{q}) \\ H_2(q,\dot{q}) \end{bmatrix} \end{bmatrix},$$

$$g(x) = \begin{bmatrix} 0 \\ 0 \\ -M^{-1}(q_2)\begin{bmatrix} 0 \\ nK_{DC} \end{bmatrix} \end{bmatrix}.$$

The purpose of this paper is to design a single nonlinear optimal feedback controller for swinging up and stabilizing the Acrobot from the stable equilibrium point $x = [\pi, 0, 0, 0]$ to the unstable equilibrium point $x = [0, 0, 0, 0]$. The system parameters are shown in Table.1.

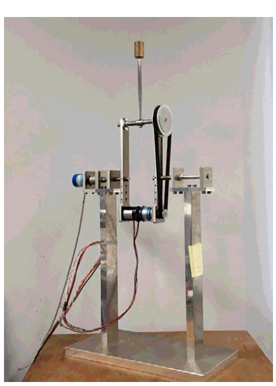

Fig. 1. Acrobot experiment devise

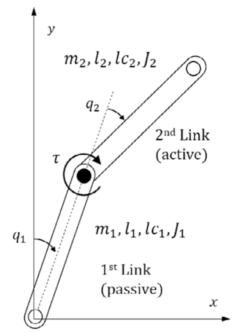

Fig. 2. Acrobot model

Table 1. System parameters

m_1	Arm1 weight	0.850[kg]
l_1	Arm1 length	0.154[m]
l_{c1}	Arm1 length from joint to COM	-0.0189[m]
J_1	Arm1 inertia	6.25×10^{-3}[kg·m^2]
μ_1	Arm1 viscous resistance coefficient	5.50×10^{-3}[N·m·s]
m_2	Arm2 weight	0.420[kg]
l_2	Arm2 length	0.210[m]
l_{c2}	Arm2 length from joint to COM	0.0743[m]
J_2	Arm2 inertia	4.48×10^{-3}[kg·m^2]
μ_2	Arm2 viscous resistance coefficient	0.0160[N·m·s]
g	Acceleration of gravity	9.801[m/s^2]
n	Gear ratio	14:48
K_{DC}	Electromotive torque constant	0.0160[N·m/V]

3. HAMILTON-JACOBI EQUATION AND STABLE MANIFOLD

In this section, we briefly review the stable manifold method proposed in Sakamoto and van der Schaft (2008); Sakamoto (2013) for numerically solving HJEs for nonlinear optimal control problem. It is first shown that a stable manifold of the Hamiltonian system associated with a HJE is equivalent to the stabilizing solution of the HJE (I). Then, the stable manifold method algorithm to compute flows on the stable manifold of the Hamiltonian system is presented (II). Finally, the optimal state feedback function is constructed from the flow data by polynomial functions that define the stable manifold (III). In what follows, the procedures (I)∼(III) will be described with some details.

Let us consider the optimal regulation problem for the following nonlinear affine system and quadratic cost function J.

$$\begin{cases} \dot{x} = f(x) + g(x)u,\ x(0) = x_0 \\ J = \dfrac{1}{2}\displaystyle\int_0^\infty \left(x^TQx + u^TRu\right) dt, \end{cases} \quad (3)$$

where $x \in \mathbb{R}^n$, $u \in U \in \mathbb{R}^m$ and $R \in \mathbb{R}^{n\times m}$, $Q \in \mathbb{R}^{n\times m}$ are positive-definite matrices. We also assume that $f(\cdot) : \mathbb{R}^n \to \mathbb{R}^n$ with $f(0) = 0$ and $g(\cdot) : \mathbb{R}^n \to \mathbb{R}$ are all C^∞. It is then possible to write $f(x) = Ax + o(|x|)$, $g(x) = B + O(|x|)$ with real matrices $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$.

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