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Decentralized Adaptive Coverage Control of Nonholonomic Mobile Robots ⋆

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Abstract: This paper proposes an algorithm for decentralized adaptive control for coverage of a given convex region using multiple agents with double integrator dynamics. The result is then extended to nonholonomic mobile robots. The density function which describes the event of interest to be sensed by the agents is assumed to be unknown and an adaptation law is derived which can learn the sensory function with time. This work is an extension of the work by Schwager et al. (2007) for mobile sensors with single integrator dynamics. We propose an algorithm which ensures convergence to a near optimal configuration and demonstrate the same using simulations.

Keywords: coverage control; multi-agent; adaptive control; decentralized control;

1. INTRODUCTION

In the past few years, there has been significant research on cooperative control problems where multiple entities or agents cooperate to perform a single overall task (Jadbabaie and Lin (2003), Murray (2007), Olfati-Saber et al. (2007), Tanner et al. (2007)). The applications are mostly targeted towards mobile robots, unmanned aerial vehicles (UAVs) etc. Decentralized and distributed control schemes are used for effectively accomplishing cooperative control objectives. Coverage is one such cooperative control problem where multiple agents are deployed to cover a given area for sensing or some other purpose.

In Cortes et al. (2004), the authors have proposed algorithms for decentralized coverage control of multiple agents deployed to cover a convex region in euclidean space. The sensory function indicating the relative importance of different locations in the convex region may not be known a priori. In Schwager et al. (2007), the authors provide an adaptive algorithm for decentralized coverage control where the sensory function is assumed to be unknown but static. The algorithm was derived for agents with first order dynamics. In Luna et al. (2010), the authors extend the work to nonholonomic agent kinematics. We consider the second order dynamic models of the mobile agents which includes the kinematic equations. We first derive the control and adaptation law for simple double integrator agents and then extend the same to more realistic dynamic model of mobile robots as given in Fierro and Lewis (1997).

In section 2, we discuss the coverage problem in more detail. In sections 3 and 4, we discuss adaptive control algorithms for coverage using double integrator agents and nonholonomic mobile robots. In section 5, we present some simulation results obtained using the algorithms presented and finally we conclude the paper with section 6.

2. PROBLEM DESCRIPTION

We consider a convex region $\mathcal{Q} \subset \mathbb{R}^q$ in which n agents are deployed to cover the region. The position of each agent will be denoted by $p_i \in \mathbb{R}^q$ and the corresponding velocities are denoted by $\dot{p}_i$ for each $i \in \{1, 2, \ldots, n\}$. The coverage problem is formulated as in Cortes et al. (2004). We consider a density function $\phi : \mathcal{Q} \to \mathbb{R}_+$ over $\mathcal{Q}$ which describes the relative importance of various regions of $\mathcal{Q}$ with respect to coverage objective i.e. the regions where ϕ has higher values are more important than the regions with lower values of ϕ and in the optimal coverage configuration, the agents should cover the region in proportion to the value of ϕ.

The voronoi partitions generated by a set of points $\{p_1, p_2, \ldots p_n\}$ is defined as

$$\mathcal{V}_i = \{q : \ ||q - p_i|| \leq ||q - p_j|| \quad \forall j \in \{1, 2 \ldots n\}, j \neq i\} \tag{1}$$

We assume that each agent at position p_i covers a region W_i. We also assume that the sensing reliability of a point in the domain by an agent decreases with the distance of the point from the agent's location. We can then formulate the following cost function [Cortes et al. (2004),Bullo et al. (2009)]:

$$\mathcal{H}(p_1, \ldots, p_n, \mathcal{W}_1, \ldots, \mathcal{W}_n) = \sum_{i=1}^{n} \int_{\mathcal{W}_i} ||q - p_i||^2 \phi(q) dq \tag{2}$$

The optimal coverage configuration is then the set of agent positions p_i and the corresponding coverage region of agents W_i such that the cost in equation (2) is minimized.

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The cost function (2) is minimized when the coverage region of the agents $\mathcal{W}_i$ correspond to the Voronoi partitions $\mathcal{V}_i$ generated by the agent positions $\mathcal{P} = \{p_1, p_2, \ldots, p_n\}$ (see Du et al. (1999)). Then,

$$\mathcal{H}(p_1, \ldots, p_n) = \sum_{i=1}^{n} \int_{\mathcal{V}_i} ||q - p_i||^2 \phi(q) dq \tag{3}$$

It can also be shown that the gradient of $\mathcal{H}$ with respect to the agent positions p_i (see Cortes et al. (2002),Cortes et al. (2004),Du et al. (1999)) is given by

$$\frac{\partial \mathcal{H}}{\partial p_i} = -M_{\mathcal{V}_i}(C_{\mathcal{V}_i} - p_i) \tag{4}$$

where

$$L_{\mathcal{V}_i} = \int_{\mathcal{V}_i} q\phi(q) dq \tag{5}$$

$$M_{\mathcal{V}_i} = \int_{\mathcal{V}_i} \phi(q) dq \tag{6}$$

$$C_{\mathcal{V}_i} = \frac{L_{\mathcal{V}_i}}{M_{\mathcal{V}_i}} \tag{7}$$

$M_{\mathcal{V}_i}$ is called the mass of $\mathcal{V}_i$, $L_{\mathcal{V}_i}$ is the first mass-moment of $\mathcal{V}_i$ and $C_{\mathcal{V}_i}$ is the centroid of $\mathcal{V}_i$. From equation (4), we see that the cost function (3) achieves local minimum when $p_i = C_{\mathcal{V}_i}$, i.e. location of each agent corresponds to the centroid of its voronoi partition. We call such a configuration the *centroidal voronoi configuration*.

Thus we seek to obtain control laws for the agents to cover the region $\mathcal{Q}$ optimally by making the agents converge to a centrodial voronoi configuration.

3. COVERAGE FOR DOUBLE INTEGRATOR AGENTS

We consider the following dynamics for the individual agents:

$$\ddot{p}_i = u_i \qquad i = 1, 2, \ldots, n. \tag{8}$$

This can be rewritten as

$$\dot{p}_{i_1} = p_{i_2} \tag{9}$$

$$\dot{p}_{i_2} = u_i \tag{10}$$

where $p_{i_1} := p_i$ is the position of agent i and $p_{i_2} := \dot{p}_i$ is the velocity of agent i.

The agents are deployed to cover the region $\mathcal{Q}$ and it is assumed that the density function $\phi(q)$ is unknown to the agents. It is also assumed that the density function can be written in the form

$$\phi(q) = \mathcal{K}(q)^T a \tag{11}$$

where $\mathcal{K} : \mathbb{R}^q \to \mathbb{R}_+^m$ and $a \in \mathbb{R}_+^m$ is a constant vector which we will call the parameter vector. It is assumed that $\mathcal{K}(q)$ is known to all the agents but a is unknown. $\mathcal{K}(q)^T = [\mathcal{K}_1(q), \mathcal{K}_2(q), \ldots, \mathcal{K}_m(q)]$ can be interpreted as a set of basis functions whose weighted combination gives the density function $\phi(q)$. The parameter vector a is assumed to be lower bounded and thus satisfies

$$a(i) \geq a_{\min} \qquad i = 1, 2, \ldots, m \tag{12}$$

where $a(i)$ is the ith component of a. This ensures that $\phi(q)$ never becomes zero which will lead to $C_{\mathcal{V}_i}$ being undefined.

It is assumed that each agent can measure the value of density function $\phi(q)$ at its current location. Since the true parameter a is not known, each agent will use an estimate of a denoted by $\hat{a}_i$. An adaptation law will be derived for $\hat{a}_i$ so that the desired objectives are achieved. We also define the corresponding estimated quantities $\hat{\phi}_i(q) = \mathcal{K}(q)^T \hat{a}_i$ which is the agent i's estimate of the density function, $\hat{M}_{\mathcal{V}_i} = \int_{\mathcal{V}_i} \hat{\phi}_i(q) dq$ which is the agent i's estimate of the mass of $\mathcal{V}_i$, $\hat{L}_{\mathcal{V}_i} = \int_{\mathcal{V}_i} q\hat{\phi}_i(q) dq$ which is the agent i's estimate of $L_{\mathcal{V}_i}$ and $\hat{C}_{\mathcal{V}_i} = \frac{\hat{L}_{\mathcal{V}_i}}{\hat{M}_{\mathcal{V}_i}}$ which is agent i's estimate of the centroid of $\mathcal{V}_i$.

Theorem 1 below gives an adaptive decentralized control law for agents with dynamics (10) to converge to a centroidal voronoi configuration under the above stated assumptions. The control law is given by

$$u_i = -k_1 \hat{M}_{\mathcal{V}_i}(p_{i_1} - \hat{C}_{\mathcal{V}_i}) - k_2 p_{i_2} \tag{13}$$

$$= -k_1 \hat{M}_{\mathcal{V}_i}(p_i - \hat{C}_{\mathcal{V}_i}) - k_2 \dot{p}_i \tag{14}$$

In addition the following quantities are defined, [Schwager et al. (2007)],

$$\Lambda_i(t) = \int_0^t e^{-\alpha(t-\tau)} \mathcal{K}_i(\tau) \mathcal{K}_i(\tau)^T d\tau \tag{15}$$

$$\lambda_i(t) = \int_0^t e^{-\alpha(t-\tau)} \mathcal{K}_i(\tau) \phi_i(\tau) d\tau \tag{16}$$

$$b_i = -k_1 \int_{\mathcal{V}_i} \mathcal{K}(q)(q - p_i)^T dq \, p_{i_2} - \gamma(\Lambda_i \hat{a}_i - \lambda_i) \tag{17}$$

where $\mathcal{K}_i(\tau) := \mathcal{K}(p_{i_1}(t))$, $\phi_i = \phi(p_{i_1}(t))$ which corresponds to agent i's measurement of the density function $\phi(q)$, α and γ are positive constants.

$\Lambda_i(t)$ and $\lambda_i(t)$ can be obtained using the following filter equations with zero initial conditions.

$$\dot{\Lambda}_i = -\alpha \Lambda_i + \mathcal{K}_i \mathcal{K}_i^T$$

$$\dot{\lambda}_i = -\alpha \lambda_i + \mathcal{K}_i \phi_i$$

Then the adaptive law for $\hat{a}_i$ is given by

$$\dot{\hat{a}}_i = \Gamma(b_i - I_{\beta_i} b_i) \tag{18}$$

with

$$I_{\beta_i} = \begin{cases} 0 & \text{for } \hat{a}_i(j) > a_{\min} \\ 0 & \text{for } \hat{a}_i(j) = a_{\min} \,\&\, b_i(j) \geq 0 \\ 1 & \text{otherwise} \end{cases} \tag{19}$$

and $\Gamma > 0$ is positive definite. The adaptation law consists of the term b_i in addition to a projection defined by (19) to make sure that the updated parameter value is always greater than the assumed minimum $a_{\min}$.
We now state the theorem.

Theorem 1. Consider n agents with dynamics given by (10). With the control law given by (14) and the update law for $\hat{a}_i$ given by (18), the following holds:
(a) $\lim_{t\to\infty}(p_{i_1} - \hat{C}_{\mathcal{V}_i}) = 0 \quad \forall i \in \{1, 2, \ldots, n\}$.
(b) $\lim_{t\to\infty} \dot{p}_{i_2} = 0 \quad \forall i \in \{1, 2, \ldots, n\}$
(c) $\lim_{t\to\infty} \mathcal{K}_i(\tau)\tilde{a}_i(t) = 0 \quad \forall \tau$ s.t $0 \leq \tau \leq t$ and $\forall i \in \{1, 2, \ldots, n\}$.

Proof. The proof follows along similar lines as the proof of the main theorem in Schwager et al. (2007). Define the modified potential function

$$V(t) = k_1 \mathcal{H} + \frac{1}{2} \sum_{i=1}^{n} \tilde{a}_i^T \Gamma^{-1} \tilde{a}_i + \frac{1}{2} \sum_{i=1}^{n} p_{i_2}^T p_{i_2} \tag{20}$$