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Global sliding-mode observers for a class of mechanical systems with disturbances ⋆

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Abstract: The problem of velocity estimation in a mechanical system has been paid a lot of attention for the past several years. In this paper we propose a finite-time observer for a class of non-linear systems containing the Lagrangian systems with disturbances. The discontinuous observer design, proposed here, does not need bounded trajectories in the system, as opposed to what is considered in the literature. To deal with the quadratic term in the velocity, a transformation of states is introduced. Dissipative properties are used for observer design and this is reduced to the solution of some matrix inequalities.

Keywords: Velocity observers; Non-linear observers; Sliding-mode observers; Dissipative observers.

1. INTRODUCTION

The control of mechanical systems requires both position and velocity. Since usually only position is measured, it is necessary to estimate the velocity by means of an observer. When the non-linear model, the parameters and the inputs of the system are well known there is an extensive literature providing global and asymptotically converging velocity estimation, see e.g. (Gauthier et al., 1992; Besançon, 2000, 2007; Astolfi et al., 2010). However, in the presence of uncertainties, discontinuous nonlinearities (e.g. dry friction) and/or unknown inputs (unknown torque) the challenge in estimating globally, exactly and in finite time the value of the velocity becomes more difficult. In particular, it is impossible to estimate the velocity in the presence of an *arbitrary* unknown input (UI), since the relative degree of the UI with respect to the measured position is two, instead of one, as it is required by the UI observer theory (Hautus, 1983; Rocha-Cózatl and Moreno, 2004, 2011). To allow the estimation of the velocity in this paper we assume that the UI is *bounded*.

For this purpose a discontinuous estimation algorithm is required, such as a sliding-mode observers (Pisano and Usai, 2011; Edwards et al., 2002; Spurgeon, 2008; Barbot et al., 2003). One of their advantages is that they provide theoretically exact convergence to the true system's states (Fridman et al., 2008; Barbot and Floquet, 2010; Bejarano et al., 2011; Efimov et al., 2012), even in the presence of *bounded unknown inputs*, under the condition that the non-linear system has a Bounded-Input-Bounded-State (BIBS) property with respect to the unknown input/disturbance. In particular, for observation of non-linear mechanical systems with disturbances, most papers using sliding-mode methodology (Xian et al., 2004; Davila et al., 2005) require the system to be BIBS. To overcome this restriction for linear systems in (Fridman et al., 2007; Barbot and Floquet, 2010; Fridman et al., 2011) and for a class nonlinear systems (Apaza-Perez et al., 2015) the authors propose the following strategy: (i) a Luenberger observer that estimates the (possibly unbounded) states of the system, despite the presence of the bounded unknown input, so that the estimation error converges to a neighborhood of zero. (ii) In order to obtain the exact value of the state in finite-time a higher-order sliding-mode differentiator is applied to the output estimation error of the Luenberger observer. (iii) A combination of the signals provided by both algorithms gives indeed the exact value of the states of the plant after finite-time, even for systems that do not fulfill the BIBS property.

For observation of non-linear systems a dissipative approach proposed in (Moreno, 2004, 2005) for systems with known inputs, and in (Rocha-Cózatl and Moreno, 2004, 2011) for UI observers, results to be efficient. This technique contains as particular cases well-known observer design methods, as e.g. Lipschitz (Rajamani, 1998) and high gain observers (Gauthier et al., 1992). For systems with UI satisfying the conditions for existence of an UI observer, among them the relative degree one condition, the dissipative observer is able to estimate globally and exponentially the true states (Rocha-Cózatl and Moreno, 2004, 2011). When the relative degree is higher than one but the UI is bounded the dissipative observer assures the convergence to a neighborhood of the origin of the

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estimation error (Moreno, 2005).

Problem statement. Consider the following second-order system, that can represent a class of mechanical systems:

$$\begin{cases} \dot{\xi}_1 = \xi_2, \\ \dot{\xi}_2 = \psi(\xi_1, \xi_2) + \alpha(\xi_1)\xi_2^2 + u + F_t + w_1(t, \xi_1, \xi_2), \\ y = \xi_1, \end{cases} \tag{1}$$

where $\xi_1, \xi_2 \in \mathbb{R}$ are the states; $y \in \mathbb{R}$ is the measured output; $\psi(\cdot)$ is a non-linear continuous function and $\alpha(\cdot)$ is a continuous function; $u \in \mathbb{R}$ is the control input; F_t is the discontinuous signal (e.g. effect due to dry friction); $w_1(t, \xi_1, \xi_2) \in \mathbb{R}$ includes other bounded unknown signals (e.g. hysteresis phenomenon, external disturbances, etc.). We assume that the system (1) is forward complete, i.e. the trajectories are defined for all positive time.

The objective is to design an observer that estimates globally, exactly and in finite time the unmeasured state despite the presence of UI.

Main contribution. We propose an observer design for a class of non-linear systems containing the Lagrangian systems with disturbances, this disturbances are not necessary vanishing. The unmeasured state is estimated in finite time despite the presence of discontinuous terms and disturbances. The BIBS property of the system is not required for our design.

The rest of the paper is organized as follows. Section 2 contains a motivating example showing that the sliding-mode differentiator cannot ensure the design of a finite-time observer. A states transformation for the system (1) and its application to Lagrangian systems, is described in Section 3. In Section 4, the observer design is analyzed. The main results, for the case with and without disturbances, is showing in Section 5. Section 6 provides some conclusions.

2. BACKGROUND AND NOTATION

Throughout this paper we make use of the following notations:

$$\begin{aligned} sign(\cdot):&\quad \text{sign function}, \\ \lceil\cdot\rfloor^p :=&\ |\cdot|^p sign(\cdot), \\ \begin{bmatrix} \cdot & \star \\ \cdot & \cdot \end{bmatrix}:&\quad \text{symmetric matrix}. \end{aligned}$$

Definition 1. (Rocha-Cózatl and Moreno (2011)). For a non-linear system with input $u \in \mathbb{R}^p$ and output $y \in \mathbb{R}^m$.

- A function $\omega : \mathbb{R}^m \times \mathbb{R}^p \to \mathbb{R}$ such that $\omega(0,0) = 0$, is called *supply rate* for the system, if $\omega(y,u)$ is locally integrable for all input-output pair of the system; i.e.
$$\int_{t_0}^{t_1} |\omega(y(s), u(s))| ds < \infty, \quad \forall t_1, t_0 \in \mathbb{R}, \ \ t_0 \le t_1.$$
- A non-linearity time variant $\gamma : [0,\infty) \times \mathbb{R}^p \to \mathbb{R}^m$, $\gamma(t,u) = y$, piecewise continuous in t, locally Lipschitz in u such that $\gamma(t,0) = 0$ is called *dissipative regarding the supply rate* $\omega(y,u)$, if for each $t \ge 0$ and $u \in \mathbb{R}^p$
$$\omega(\gamma(t,u), u) \ge 0.$$

If the supply rate $\omega(y,u)$ is a quadratic form: $\omega(y,u) = \begin{bmatrix} y \\ u \end{bmatrix}^T \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \begin{bmatrix} y \\ u \end{bmatrix}$, where $Q \in \mathbb{R}^{m\times m}$, $S \in \mathbb{R}^{m\times p}$, $R \in \mathbb{R}^{p\times p}$ and Q and R are symmetrical, we will abbreviate it by $\{Q,S,R\}-$dissipative.

The generalized super-twisting algorithm (Moreno, 2009) as differentiator for estimating the derivative of the function $f(\cdot)$, with second derivative $\ddot{f}(t)$ bounded, is given by

$$\begin{cases} \dot{z}_1(t) = -k_1\left(\mu_1\lceil e(t)\rfloor^{1/2} + \mu_2\lceil e(t)\rfloor\right) + z_2(t) \\ \dot{z}_2(t) = -k_2\left(\dfrac{\mu_1^2}{2}\lceil e(t)\rfloor^0 + \dfrac{3\mu_1\mu_2}{2}\lceil e(t)\rfloor^{1/2} + \mu_2^2\lceil e(t)\rfloor\right) \end{cases} \tag{2}$$

where $e(t) = z_1(t) - f(t)$, and μ_1, μ_2, k_1 and k_2 are design parameters. This algorithm is inside the sliding-mode technique.

2.1 Motivating example

The following example illustrates the loss of effectiveness of the generalized super-twisting algorithm based differentiator. We consider the example given in (Besançon, 2000),

$$\underbrace{(1 + cos^2(q))}_{m(q)}\ddot{q} \underbrace{-\frac{1}{2}\sin(2q)}_{c(q)}\dot{q}^2 + g\sin(q) = \tau, \tag{3}$$

where the condition $\dot{m}(q) = 2c(q)\dot{q}$, is satisfied. This condition allows them to cancel the quadratic dependence on q through a state transformation. If $w_1(t) = \tau$ is an unknown input with a slightly change in the coefficient of the quadratic term, and introducing a discontinuous term (the dry friction) and a term that avoids the BIBS property on the system

$$(1+cos^2(q))\ddot{q} - \frac{1}{2}\sin(q)\dot{q}^2 + g\sin(q) - \frac{\cos^2(0.5q)}{2}\dot{q} + 0.5sign(\dot{q}) = w_1(t). \tag{4}$$

For this system the condition $\dot{m}(q) = 2c(q)$ is not satisfied such that the proposed observer in (Besançon, 2000) cannot be used.

Rewriting this system (4) with $\xi_1 = q$, $\xi_2 = \dot{q}$ as

$$\begin{cases} \dot{\xi}_1 = \xi_2, \\ \dot{\xi}_2 = \dfrac{\cos^2(0.5\xi_1)}{2(1+\cos^2(\xi_1))}\xi_2 + \dfrac{\sin(\xi_1)}{2(1+\cos^2(\xi_1))}\xi_2^2 + \\ \qquad + \dfrac{-9.8\sin(\xi_1) - 0.5sign(\xi_2) + w_1(t)}{1+\cos^2(\xi_1)}. \end{cases} \tag{5}$$

For the simulations, we consider the unknown input as

$$w_1(t) = 0.5\sin(3t)\cos(t) - 1.5\cos(\pi t) + 1, \tag{6}$$

and the initial conditions as $(\xi_1(0), \xi_2(0)) = (-0.5, 0.5)$.

Applying the sliding-mode differentiator (2) with parameters $k_1 = -1.5\sqrt{20}$, $k_2 = -2.2(20)$, $\mu_1 = 1$ y $\mu_2 = 0.5$ to the previous example (5). We get the following simulations with the dynamical system

$$\begin{aligned} \dot{z}_1 =& -6.71\lceil e_1\rfloor^{1/2} - 3.35\lceil e_1\rfloor + z_2, \\ \dot{z}_2 =& -22\lceil e_1\rfloor^0 - 33\lceil e_1\rfloor^{1/2} + 11\lceil e_1\rfloor \end{aligned} \tag{7}$$

where $e_1 = z_1 - \xi_1$ and initial conditions $z_1(0) = z_2(0) - 30$.