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Adaptive Optimal PMU Placement Based on Empirical Observability Gramian [⋆]

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Abstract: In this paper, we compare four measures of the empirical observability gramian, including the determinant, the trace, the minimum eigenvalue, and the condition number, which can be used to quantify the observability of system states and to obtain the optimal PMU placement for power system dynamic state estimation. An adaptive optimal PMU placement method is proposed by automatically choosing proper measures as the objective function. It is shown that when the number of PMUs is small and thus the observability is very weak, the minimum eigenvalue and the condition number are better measures of the observability and are preferred to be chosen as the objective function. The effectiveness of the proposed method is validated by performing dynamic state estimation on an Northeast Power Coordinating Council (NPCC) 48-machine 140-bus system with the square-root unscented Kalman filter.

Keywords: Adaptive, condition number, determinant, dynamic state estimation, empirical observability gramian, optimal PMU placement (OPP), phasor measurement unit (PMU), smallest eigenvalue, square-root unscented Kalman filter, synchrophasor, trace.

1. INTRODUCTION

Increasing integration of intermittent renewable energy and current effort of developing smart grid will make electric power systems more and more dynamic. However, the most widely studied power system static state estimation (SSE) (Schweppe-Wildes (1970); Abur (2004); Monticelli (2000); Irving (2008); He (2011); Qi (2012)) cannot capture the dynamics of power systems well due to its dependency on slow update rates of Supervisory Control and Data Acquisition (SCADA) systems.

By contrast, real-time dynamic state estimation (DSE) enabled by phasor measurement units (PMUs), which has high update rates and high global positioning system (GPS) synchronization accuracy, can provide accurate dynamic states of the system and thus will play a critical role in achieving real-time wide-area monitoring, protection, and control (Begovic (2005); Qi (2015b)). Until now DSE has been implemented by extended Kalman filter (Huang (2007); Ghahremani (2011)), unscented Kalman filter (Wang (2012); Singh (2014)), square-root unscented Kalman filter (Qi (2015a,c)), extended particle filter (Zhou (2013)), cubature Kalman filter(Qi (2016a)), and observers (Taha (2015); Qi (2016a)).

The well-known optimal PMU placement (OPP) problem was originally developed for SSE. It is mainly based on the topological observability criterion, which only specifies that the power system states should be uniquely estimated with the minimum number of PMUS but neglects important parameters such as transmission line admittances by only focusing on the binary connectivity graph (Baldwin (1993); Li (2013)). Under this framework, many approaches have been proposed, such as mixed integer programming (Xu (2004); Gou (2008)), binary search (Chakrabarti (2008a)), metaheuristics (Milosevic (2003); Aminifar (2009)), particle swarm optimization (Chakrabarti (2008b)), and eigenvalue-eigenvector based approaches (Almutairi (2009); Korba (2003)). An information-theoretic criterion is also proposed to generate highly informative PMU configurations (Li (2013)).

However, not much research has been done on OPP for DSE. In (Kamwa (2002)) numerical PMU configuration algorithms are proposed to maximize the overall sensor response while minimizing the correlation among sensor outputs based on the system response under many contingencies. In (Sun (2011)) an OPP strategy is proposed to ensure a satisfactory state tracking performance but it depends on a specific Kalman filter. In (Qi (2015c)) the empirical observability gramian (Lall (1999, 2002); Hahn (2002); Singh (2006)) is applied to quantify the degree of observability of the system states and OPP is achieved by maximizing the determinant of the empirical observability gramian. Compared with (Kamwa (2002)) and (Sun (2011)), (Qi (2015c)) has a quantitative mea-

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sure of observability, which makes it possible to optimize PMU locations from the point view of the observability of nonlinear systems. Besides, it only needs to deal with the system under typical power flow conditions and also does not depend on the specific realization of any Kalman filter.

As in (Qi (2015c)), there are various measures of the empirical observability gramian that can be used to quantify the observability of the system state. In this paper, we compare these measures and further propose an adaptive OPP method for power system dynamic state estimation by automatically choosing proper measures as the objective function to guarantee best observability.

The remainder of this paper is organized as follows. Section 2 briefly introduces power system dynamic state estimation. Section 3 discusses the definition and implementation of empirical observability gramian. Section 4 introduces the OPP method based on the empirical observability gramian and proposes an adaptive OPP method by making full use of different measures. The results are presented in Section 5 in order to test and validate the proposed method. Finally the conclusion is drawn in Section 6.

2. POWER SYSTEM DYNAMIC STATE ESTIMATION

Different from SSE, DSE estimates the dynamic states (internal states of generators), rather than the static states (voltage magnitude and phase angles of buses). In order to perform DSE, the nonlinear dynamics and the outputs of a power system is described in the following form:

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}) \\ \boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{u}) \end{cases} \tag{1}$$

where $\boldsymbol{f}(\cdot)$ and $\boldsymbol{h}(\cdot)$ are the state transition and output functions, $\boldsymbol{x} \in \mathbb{R}^n$ is the state vector, $\boldsymbol{u} \in \mathbb{R}^v$ is the input vector, and $\boldsymbol{y} \in \mathbb{R}^p$ is the output vector.

We consider two types of generator models, the fourth-order transient model and second-order classical model. For generator with transient model, the fast sub-transient dynamics and saturation effects are ignored and the generator is described by fourth-order differential equations:

$$\begin{cases} \dot{\delta}_i = \omega_i - \omega_0 \\ \dot{\omega}_i = \dfrac{\omega_0}{2H_i}\Big(T_{mi} - T_{ei} - \dfrac{K_{Di}}{\omega_0}(\omega_i - \omega_0)\Big) \\ \dot{e}'_{qi} = \dfrac{1}{T'_{d0i}}\Big(E_{fdi} - e'_{qi} - (x_{di} - x'_{di})\, i_{di}\Big) \\ \dot{e}'_{di} = \dfrac{1}{T'_{q0i}}\Big(-e'_{di} + (x_{qi} - x'_{qi})\, i_{qi}\Big) \end{cases} \tag{2}$$

where i is the generator serial number, δ_i is the rotor angle, ω_i is the rotor speed in rad/s, and e'_{qi} and e'_{di} are the transient voltage along q and d axes; i_{qi} and i_{di} are stator currents at q and d axes; T_{mi} is the mechanical torque, T_{ei} is the electric air-gap torque, and E_{fdi} is the internal field voltage; ω_0 is the rated value of angular frequency, H_i is the inertia constant, and K_{Di} is the damping factor; T'_{q0i} and T'_{d0i} are the open-circuit time constants for q and d axes; x_{qi} and x_{di} are the synchronous reactance and x'_{qi} and x'_{di} are the transient reactance at the q and d axes.

Generators with classical model are described by the first two equations of (2) and e'_{qi} and e'_{di} are kept unchanged.

T_{mi} and E_{fdi} are considered as inputs and are assumed to be constant and known. Let $\mathcal{G}_P$ denote the set of generators where PMUs are installed. For generator $i \in \mathcal{G}_P$, the terminal voltage phaosr $E_{ti} = e_{Ri} + je_{Ii}$ and the terminal current phasor $I_{ti} = i_{Ri} + ji_{Ii}$ can be measured, and are used as the outputs.

The dynamic model (2) can be rewritten in a general state space form in (1) and the state vector $\boldsymbol{x}$, input vector $\boldsymbol{u}$, and output vector $\boldsymbol{y}$ can be written as

$$\boldsymbol{x} = \Big[\boldsymbol{\delta}^\top \quad \boldsymbol{\omega}^\top \quad \boldsymbol{e'_q}^\top \quad \boldsymbol{e'_d}^\top\Big]^\top \tag{3}$$

$$\boldsymbol{u} = \big[\boldsymbol{T_m}^\top \quad \boldsymbol{E_{fd}}^\top\big]^\top \tag{4}$$

$$\boldsymbol{y} = \big[\boldsymbol{e_R}^\top \quad \boldsymbol{e_I}^\top \quad \boldsymbol{i_R}^\top \quad \boldsymbol{i_I}^\top\big]^\top. \tag{5}$$

The i_{qi}, i_{di}, and T_{ei} in (2) are actually functions of $\boldsymbol{x}$:

$$\begin{aligned} \Psi_{Ri} &= e'_{di}\sin\delta_i + e'_{qi}\cos\delta_i \\ \Psi_{Ii} &= e'_{qi}\sin\delta_i - e'_{di}\cos\delta_i \\ I_{ti} &= \overline{\boldsymbol{Y}}_i(\boldsymbol{\Psi}_R + j\,\boldsymbol{\Psi}_I) \\ i_{Ri} &= \mathrm{Re}(I_{ti}) \\ i_{Ii} &= \mathrm{Im}(I_{ti}) \\ i_{qi} &= \frac{S_B}{S_{Ni}}(i_{Ii}\sin\delta_i + i_{Ri}\cos\delta_i) \\ i_{di} &= \frac{S_B}{S_{Ni}}(i_{Ri}\sin\delta_i - i_{Ii}\cos\delta_i) \\ e_{qi} &= e'_{qi} - x'_{di}i_{di} \\ e_{di} &= e'_{di} + x'_{qi}i_{qi} \\ P_{ei} &= e_{qi}i_{qi} + e_{di}i_{di} \\ T_{ei} &= \frac{S_B}{S_{Ni}}P_{ei} \end{aligned}$$

where $\Psi_i = \Psi_{Ri} + j\Psi_{Ii}$ is the voltage source, $\boldsymbol{\Psi}_R$ and $\boldsymbol{\Psi}_I$ are column vectors of all generators' Ψ_{Ri} and Ψ_{Ii}, e_{qi} and e_{di} are the terminal voltage at q and d axes, and $\overline{\boldsymbol{Y}}_i$ is the ith row of the admittance matrix of the reduced network $\overline{\boldsymbol{Y}}$ whose elements are constant if the difference between x'_d and x'_q is ignored (Wang (2015)), P_{ei} is the electrical active output power, and S_B and S_{Ni} are the system base MVA and the base MVA for generator i, respectively.

The outputs i_R and i_I have been written as functions of $\boldsymbol{x}$. Similarly, the outputs e_{Ri} and e_{Ii} can also be written as function of $\boldsymbol{x}$:

$$\begin{aligned} e_{Ri} &= e_{di}\sin\delta_i + e_{qi}\cos\delta_i \\ e_{Ii} &= e_{qi}\sin\delta_i - e_{di}\cos\delta_i. \end{aligned}$$

The continuous model in (1) can be discretized as

$$\begin{cases} \boldsymbol{x}_k = \boldsymbol{f}_d(\boldsymbol{x}_{k-1}, \boldsymbol{u}_{k-1}) \\ \boldsymbol{y}_k = \boldsymbol{h}(\boldsymbol{x}_k, \boldsymbol{u}_k) \end{cases} \tag{6}$$

where the state transition functions $\boldsymbol{f}_d$ can be obtained by the modified Euler method as

$$\tilde{\boldsymbol{x}}_k = \boldsymbol{x}_{k-1} + \boldsymbol{f}(\boldsymbol{x}_{k-1}, \boldsymbol{u}_{k-1})\Delta t \tag{7}$$

$$\tilde{\boldsymbol{f}} = \frac{\boldsymbol{f}(\tilde{\boldsymbol{x}}_k, \boldsymbol{u}_k) + \boldsymbol{f}(\boldsymbol{x}_{k-1}, \boldsymbol{u}_{k-1})}{2} \tag{8}$$

$$\boldsymbol{x}_k = \boldsymbol{x}_{k-1} + \tilde{\boldsymbol{f}}\Delta t. \tag{9}$$