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Functional Observers for Nonlinear Systems

Costas Kravaris

Texas A&M University, College Station, TX 77843-3122 USA
(Tel: 979-458-4514; e-mail: kravaris@tamu.edu).

Abstract: In this work, the problem of designing observers for estimating a single nonlinear functional of the state is formulated for general nonlinear systems. Notions of functional observer linearization are also formulated, in terms of achieving exactly linear error dynamics in transformed coordinates and with prescribed rate of decay of the error. Necessary and sufficient conditions for the existence of a lower-order functional observer with linear dynamics and linear output map are derived. The results provide a direct generalization of Luenberger's linear theory of functional observers to nonlinear systems.

Keywords: Functional observers, exact linearization, observer design, chemical process applications.

1. INTRODUCTION

The problem of estimating a function of the state vector, without the need of estimating the entire state vector, arises in many applications. The design of output feedback controllers based on a state feedback design is a classical example, where it is only the state feedback function that needs to be estimated and not the entire state vector. Another important class of applications is related to the design of inferential control systems, where one output is measured and a different output, which is unmeasured, needs to be regulated to set point.

These applications motivate the development of functional observers, the aim being a reduction of dimensionality relative to a full-state observer. The notion of a functional observer was first defined in Luenberger's pioneer work on observers for linear multivariable systems. Luenberger (1966, 1971) proved that it is feasible to construct a functional observer with number of states equal to observability index minus one.

The basic theory of linear functional observers can be found in standard linear systems texts, e.g. in Chen (1984). In recent years, there has been a renewed interest in functional observers for linear systems (Tsui, 1998; Trinh et al., 2006; Darouach, 2000; Korovin et al., 2008 and 2010; Fernando et al., 2010), the goal being to find the smallest possible order of the linear functional observer.

For nonlinear systems, there have been significant developments in the theory of full-state observers, with a variety of methods and approaches. In particular, in the context of exact linearization methods (Krener and Isidori, 1983; Krener and Respondek, 1985; Kazantzis and Kravaris, 1998; Kazantzis et al., 2000; Kreisselmeier and Engel, 2003; Krener and Xiao, 2002 and 2005; Andrieu and Praly, 2006), Luenberger theory for full-state observers has been extended to nonlinear systems in a direct and analogous manner. The goal of the present work is to develop a direct generalization of Luenberger's functional observers to nonlinear systems.

In the present work, we will consider unforced nonlinear systems of the form

$$\begin{aligned}\frac{dx}{dt} &= f(x)\\ y_i &= h_i(x)\,,\ i=1,\cdots,p\\ z &= q(x)\end{aligned} \tag{1}$$

where:

$x \in \mathbb{R}^n$ is the system state
$y_i \in \mathbb{R},\ i=1,\cdots,p$ are the measured outputs
$z \in \mathbb{R}$ is the (scalar) output to be estimated

and $f:\mathbb{R}^n \to \mathbb{R}^n$, $h_i:\mathbb{R}^n \to \mathbb{R}$, $q:\mathbb{R}^n \to \mathbb{R}$ are smooth nonlinear functions. The objective is to construct a functional observer of order $\nu < n$, which generates an estimate of the output z, driven by the output measurements $y_i, i=1,\cdots,p$.

Section 2 will define the notion of functional observer for a system of the form (1) in a completely analogous manner to Luenberger's definition for linear systems. Section 3 will pose the problem of functional observer design. Section 4 will define notions of exact linearization for the functional observer problem. Section 5 will develop necessary and sufficient conditions for the solution of the total linearization problem, as well as a simple formula for the resulting functional observer. In Section 6, the results of Section 5 will be specialized to linear time-invariant systems, leading to simple and easy-to-check conditions for the design of lower-order functional observers.

2. DEFINITION OF A FUNCTIONALOBSERVER FOR A NONLINEAR DYNAMIC SYSTEM

In complete analogy to Luenberger's construction for the linear case, we seek for a smooth mapping

$$\xi = \theta(x) = \begin{bmatrix}\theta_1(x)\\ \vdots \\ \theta_\nu(x)\end{bmatrix} \tag{2}$$

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from $\mathbb{R}^n$ to $\mathbb{R}^\nu$, to immerse system (1) to a ν-th order system (ν < n), with inputs y_i, $i=1,\cdots,p$ and output z:

$$\begin{aligned}\frac{d\xi}{dt} &= \varphi(\xi, y_1,\cdots,y_p)\\ z &= \omega(\xi, y_1,\cdots,y_p)\end{aligned} \tag{3}$$

where $\varphi:\mathbb{R}^\nu\times\mathbb{R}^p\to\mathbb{R}^\nu$, $\omega:\mathbb{R}^\nu\times\mathbb{R}^p\to\mathbb{R}$, the aim being that system (3), driven by the measured outputs y_i, $i=1,\cdots,p$ of (1), can generate an estimate of unmeasured output z of (1).

But in order for system (1) to be mapped to system (3) under the mapping θ(x), the following relations have to hold:

$$\frac{\partial\theta}{\partial x}(x)f(x) = \varphi\big(\theta(x), h_1(x),\cdots,h_p(x)\big) \tag{4}$$

$$q(x) = \omega\big(\theta(x), h_1(x),\cdots,h_p(x)\big) \tag{5}$$

The foregoing considerations lead to the following definition of a functional observer:

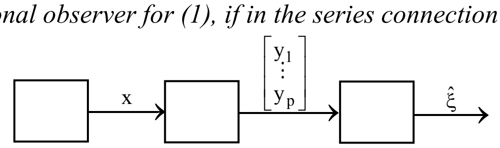
<u>Definition 1</u>: *Given a dynamic system*

$$\begin{aligned}\frac{dx}{dt} &= f(x)\\ y_i &= h_i(x)\ ,\ i=1,\cdots,p\\ z &= q(x)\end{aligned} \tag{1}$$

where $f:\mathbb{R}^n\to\mathbb{R}^n$, $h_i:\mathbb{R}^n\to\mathbb{R}$, $q:\mathbb{R}^n\to\mathbb{R}$ *are smooth nonlinear functions,* y_i, $i=1,\cdots,p$ *are the measured outputs and z is the output to be estimated , the system*

$$\begin{aligned}\frac{d\hat{\xi}}{dt} &= \varphi(\hat{\xi}, y_1,\cdots,y_p)\\ \hat{z} &= \omega(\hat{\xi}, y_1,\cdots,y_p)\end{aligned} \tag{6}$$

where $\varphi:\mathbb{R}^\nu\times\mathbb{R}^p\to\mathbb{R}^\nu$, $\omega:\mathbb{R}^\nu\times\mathbb{R}^p\to\mathbb{R}\,(\nu<n)$, *is called a functional observer for (1), if in the series connection*



the overall dynamics

$$\begin{aligned}\frac{dx}{dt} &= f(x)\\ \frac{d\hat{\xi}}{dt} &= \omega\big(\hat{\xi}, h_1(x),\cdots,h_p(x)\big)\end{aligned} \tag{7}$$

possesses an invariant manifold $\hat{\xi}=\theta(x)$ *with the property that* $q(x)=\omega\big(\theta(x),h_1(x),\cdots,h_p(x)\big)$.

In the above definition, the requirement that $\hat{\xi}=\theta(x)$ is an invariant manifold of (7), i.e. that

$$\hat{\xi}(0)=\theta(x(0)) \Rightarrow \hat{\xi}(t)=\theta(x(t)) \quad \forall t>0,$$

translates to $\left[\frac{\partial\theta}{\partial x}(x)\right]f(x) = \varphi\big(\theta(x),h_1(x),\cdots,h_p(x)\big)$,

which is condition (4) stated earlier.

If the functional observer (6) is initialized consistently with the system (1), i.e. if $\hat{\xi}(0)=\theta(x(0))$, then $\hat{\xi}(t)=\theta(x(t))$, and therefore

$$\begin{aligned}\hat{z}(t) &= \omega\big(\hat{\xi}(t), y_1(t),\cdots,y_p(t)\big)\\ &= \omega\big(\theta(x(t)), h_1(x(t)),\cdots,h_p(x(t))\big) = q(x(t)) \quad \forall t>0,\end{aligned}$$

which means that the functional observer will be able to exactly reproduce z(t).

In the presence of initialization errors, additional stability requirements will need to be imposed on the $\hat{\xi}$-dynamics, for the estimate $\hat{z}(t)$ to asymptotically converge to $z(t)$.

At this point, it is important to examine the special case of a linear system, where $f(x)=Fx$, $h_i(x)=H_ix$, $q(x)=Qx$ with F, H_i, Q being $n\times n$, $1\times n$, $1\times n$ matrices respectively, and a linear mapping $\theta(x)=Tx$ is considered. Definition 1 tells us that for a linear time-invariant system

$$\begin{aligned}\frac{dx}{dt} &= Fx\\ y_i &= H_ix\ ,\ i=1,\cdots,p\\ z &= Qx\end{aligned} \tag{8}$$

the system

$$\begin{aligned}\frac{d\hat{\xi}}{dt} &= A\hat{\xi} + \sum_{i=1}^{p} B_iy_i\\ \hat{z} &= C\hat{\xi} + \sum_{i=1}^{p} D_iy_i\end{aligned} \tag{9}$$

will be a functional observer if the following conditions are met:

$$TF = AT + \sum_{i=1}^{p} B_iH_i \tag{10}$$

$$Q = CT + \sum_{i=1}^{p} D_iH_i \tag{11}$$

for some $\nu\times n$ matrix T. These are exactly Luenberger's conditions for a functional observer for a linear time-invariant system (Luenberger, 1971).

3. DESIGNING LOWER-ORDER FUNCTIONAL OBSERVERS

For the design of a functional observer, one must be able to find a continuously differentiable map $\theta(x)=\begin{bmatrix}\theta_1(x)\\ \vdots\\ \theta_\nu(x)\end{bmatrix}$ to satisfy conditions (4) and (5), i.e. such that $\frac{\partial\theta_j}{\partial x}(x)f(x)$, $j=1,\cdots,\nu$ is a function of $\theta_1(x),\cdots,\theta_\nu(x), h_i(x)$ and $q(x)$ is a function of $\theta_1(x),\cdots,\theta_\nu(x), h_i(x)$, $i=1,\cdots,p$.