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Observabilty singularities and observer design: dual immersion approach

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Abstract It is well-known that, for nonlinear systems, the observability is often only a local property and depends on the input. Moreover, it is often required that the observer be of the same dimension as the original system. A direct consequence of this requirement is that it enlarges the set of observability singularities. If, on one hand, it is impossible to observe the state variables that are structurally unobservable, it is, however, possible to overcome the observability singularities introduced by the constraints on the observer design. In this paper, we propose a novel dual immersion method which allows to reduce the set of observability singularities. In addition, a step by step design of a high order sliding mode observer based on the proposed dual immersion approach is presented. Finally, a thorough analysis and discussion on the simulation results with respect to a non-autonomous system is given.

1. INTRODUCTION

It is a well-known that, unlike the linear case, the observability for nonlinear systems is, in general, not only a local property Hermann and Krener [1977] but also depends on the input of the system Gauthier et al. [1992]. A direct implication of these is that, in the nonlinear case, one has to determine the so-called 'observability singularity set' if proper observability analysis has to be carried out. Roughly speaking, the observability singularity set is the set of points in the state space whereby the observability matrix is not of full rank.

On the other hand, owing to the fact that it is easier to design an observer for linear systems, the natural approach to designing nonlinear observers, adopted by several researchers Krener and Respondek [1985], Xia and Gao [1989], Rudolph and Zeitz [1994], Kazantzis and Kravaris [1998], Guay [2002], Andrieu et al. [2013], Andrieu and Praly [2014], consist in transforming the original system into

i) either a linear one plus a nonlinear part having some special structures

ii) or a linear one plus a nonlinear part depending only on the input and the output so the observer has linear error dynamics.

Unfortunately, even if the observer design is done using these approaches, the problem introduced by the observability singularity set stays omnipresent and is not overcomed. In this paper, we propose a solution, based on the immersion technique, to tackle this problem under very weak conditions.

It is important to mention that, since the work of Rapaport and Maloum [2000, 2004], the immersion technique was extensively used in the observer design context. However, it is generally used to recover the linearity property by diffeomorphism and output injection Back and Soe [2004], Back et al. [2006], Ticlea [2006], Tami et al. [2013]. In the majority of the mentioned papers, immersion was realized by adding a dynamic by means of output integration Back and Soe [2004]. The stability of such extra dynamics can be problematic and an elegant solution to deal with this issue is proposed in Tami et al. [2013]. However, the problem of observability singularity was not treated. In this work, a dual immersion technique using only extra differentiations is proposed to by pass the observability singularity issue. This method is close to the one proposed in Andrieu and Praly [2014], Bernard et al. [2015] in a completely different context. Moreover, for stability reasons, we chose to employ exponentially stable dynamics instead of constant dynamics. It is shown that the proposed approach is realisable thanks to the finite time differentiator as, for example, the one proposed in Diop et al. [2000], Fliess et al. [2008] (but for these methods, the delay appears due to the data acquisition frame) or High Order Sliding Mode (HOSM) Levant [2005], Fridman et al. [2011].

The paper is organized as follows: in the next section some observability and observability singularity definitions are presented and the problem statement is explained. In Section 3, the dual immersion method is presented. After that, some recalls on HOSM differentiator are given in Section 4. In Section 5, a simulation example is given in order to highlight the fact that the proposed method can be extended to nonautonomous system. The paper ends with some conclusions and perspectives.

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2. SOME RECALLS AND PROBLEM STATEMENT

Consider the following autonomous system:

$$\begin{aligned} \dot{x} &= f(x) \\ y &= h(x) \end{aligned} \tag{1}$$

where the state $x \in \mathbb{R}^n$, the output $y \in \mathbb{R}^m$ and the vector fields f and h are assumed to be C^∞. It is also assumed that the outputs are independent for $\forall x \in \mathbb{R}^n$.

Notation: For $1 \leq i \leq m$, we denote by ρ_i the observability index Krener and Respondek [1985] of the output function h_i at $x_0 \in \mathbb{R}^n$.

It is worth noting that, if the smallest order of output derivatives are considered, then the choice of m-tuples $(\rho_1, \cdots, \rho_m)$ is not unique.

Now suppose that around a certain point x_0 the system (1) is observable, i.e. $\sum_{i=1}^m \rho_i = n$, and rank $d\mathcal{O}_{|x_0} = n$ where

$$\mathcal{O}(n) = \begin{pmatrix} h_1 \\ \vdots \\ L_f^{\rho_1 - 1} h_1 \\ \vdots \\ h_m \\ \vdots \\ L_f^{\rho_m - 1} h_m \end{pmatrix} \tag{2}$$

is the observability map and

$$d\mathcal{O}(n)_{|x_0} = \begin{pmatrix} dh_1 \\ \vdots \\ dL_f^{\rho_1 - 1} h_1 \\ \vdots \\ dh_m \\ \vdots \\ dL_f^{\rho_m - 1} h_m \end{pmatrix}_{|x_0} \tag{3}$$

is the observability matrix with $L_f^i h = \frac{\partial L_f^{i-1} h}{\partial x} f$ the usual Lie derivative and $dL_f^i h = \left(\frac{\partial L_f^i h}{\partial x_1}, \frac{\partial L_f^i h}{\partial x_2}, ..., \frac{\partial L_f^i h}{\partial x_n} \right)$ its corresponding 1-form.

Since $d\mathcal{O}(n)_{|x_0}$ is only of full rank n around x_0, this implies that there might exist some $\bar{x} \in \mathbb{R}^n$ such that rank $d\mathcal{O}(n)_{|\bar{x}} < n$. Due to this fact, we define the following observability singularity set:

$$\mathcal{S}_n = \{x \in \mathbb{R}^n : \text{rank } \{d\mathcal{O}(n)_{|x}\} < n\} \tag{4}$$

In this case, for the purpose of removing the singularities in $d\mathcal{O}(n)_{|x}$ defined in (3), it is necessary to increase the dimension of (3) by involving more derivatives of the output. The following example illustrates the procedure involved.

Example 1. Let us consider the following simple system:

$$\begin{aligned} \dot{x}_1 &= x_2 + x_2^2 \\ \dot{x}_2 &= -x_2^3 + 1 \\ \dot{x}_3 &= x_2 - x_2^3 \end{aligned} \tag{5}$$

with $y_1 = x_1$ and $y_2 = x_3$.

It can be seen that, if we choose the corresponding observability indices as $(\rho_1 = 2, \rho_2 = 1)$, then

$$d\mathcal{O}(3)_{|x} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 + 2x_2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and we obtain the following observability singularity set:

$$\mathcal{S}_3 = \{x \in \mathbb{R}^3 : x_2 = -0.5\}$$

On the other hand, if the observability indices were chosen as $(\rho_1 = 1, \rho_2 = 2)$, then

$$d\mathcal{O}(3)_{|x} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 - 3x_2^2 & 0 \end{pmatrix}$$

and the observability singularity set is now given by:

$$\mathcal{S}_3 = \{x \in \mathbb{R}^3 : x_2 = \frac{\pm 1}{\sqrt{3}}\}$$

It is clear that both observability matrices contain singularities, but they are not the same. In order to overcome those singularities, one can compute further derivatives of the output. Indeed, consider the observability indices $(\rho_1 = 3, \rho_2 = 1)$, then we obtain

$$d\mathcal{O}(4) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 + 2x_2 & 0 \\ 0 & -8x_2^3 - 3x_2^2 + 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and $\mathcal{S}_4 = \emptyset$, i.e. there is no longer any observability singularity. These highlight the fact that the choice of the observability indices is crucial in designing an observer for a nonlinear system. As a result, we obtain the following immersion from $\mathbb{R}^3$ to $\mathbb{R}^4$.

$$z = \phi(x) = \begin{pmatrix} x_1 \\ x_2 + x_2^2 \\ -2x_2^4 - x_2^3 + 2x_2 + 1 \\ x_3 \end{pmatrix}$$

The previous example shows that, even if the observability matrix $d\mathcal{O}(n)$ contains singularities, i.e. its rank is not equal to n for some $x \in \mathbb{R}^n$, it is still possible to obtain a higher dimensional map, $\mathcal{O}(n+k)$, that will not contain any singularity. This map can then be regarded as an immersion. It may, therefore, be interesting to design an observer of dimension greater than n for the system; which is equivalent to using a state space representation or order greater than n. More precisely, this immersion is obtained by finding the smallest integer $k \in \mathcal{Z}^+$ such that

$$\text{rank}\{d\mathcal{O}(n+k)\} = n, \forall x \in \mathbb{R}^n \tag{6}$$

where

$$\mathcal{O}(n+k) = \begin{pmatrix} h_1 \\ \vdots \\ L_f^{\rho_1 + k_1} h_1 \\ \vdots \\ h_m \\ \vdots \\ L_f^{\rho_m + k_m} h_m \end{pmatrix} \tag{7}$$

with $\sum_{i=1}^m \rho_i = n$ and $\sum_{i=1}^m k_i = k$. Then, the immersion can be defined as $z = \mathcal{O}(n+k)$ yielding a higher dimensional transformed system

$$\dot{z} = \frac{\partial \mathcal{O}(n+k)}{\partial x} f(x)$$