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Observer design for a class of nonlinear systems under a persistent excitation

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Abstract: The problem of state reconstruction from input and output measurements for nonlinear time delay systems remain open in many cases. In this paper we propose an adaptive observer to solve this problem for a class of unknown variable time-delay nonlinear systems where the state matrix depends on the input persistency excitation. To achieve this we combine the use of a Kalman-like observer with a suitable choice for the Lyapunov-Krasovskii functional. This is done under a sufficient number of hypothesis to guarantee the convergence of the observer inside a sphere depending of the delay upper bound. The proposed strategy is tested in simulation by considering a mixed piece-wise and sinusoid time delay function and its efficiency when the problem of persistency excitation occurs.

Keywords: Nonlinearity, Robust Observers, Time-Delay, Lyapunov Function.

1. INTRODUCTION

Time delay systems are widely used in many applications areas since time delay tends to be considered as an inherent property of many systems. This led to the investigation of an observer design for such systems in the recent years, delay can be present in the state, in the input or the output and can be constant (Germani and Pepe (2005)) or time-varying (Polyakov et al. (2013) , Assche et al. (2011)). The methods used to solve this problem consist in different observation approaches from an asymptotic approach to sliding mode, as well as many others, for both linear and non-linear systems, but many of those methods concern only a particular case. In the perspective to be more general, the delay can be considered unknown as in Lechappe et al. (2015a) and Lechappe et al. (2015b) .

Recently, this become an important center of interest (see Lechappe et al. (2015c) , Bresch-Pietri et al. (2012), Seuret et al. (2006) , Cacace et al. (2012), Krstic (2009)). In Ibrir. (2009) an observer for nonlinear systems in triangular form with variable and bounded state delay is described. The approach is an extension of known techniques (Besanon et al. (1996)) for time-varying delays, where delay is considered as a disturbance and a robust observer is developed (Germani et al. (2002) , Radke and Zhiqiang (2006) , Kazantzis and Wright (2005), Marquez-Martinez et al. (2002) , Assche et al. (2011)). Notice that in this approach only the state is estimated whereas the delay plays the role of an unknown disturbance. In other cases a robust observer might still be used, but its improvement is the use of the time delay which is identified separately. Among all the topics of time-delayed dynamical models, identification of time delays has practical importance, analogous to the significant role of parameters estimation for dynamical systems described by ordinary differential equations. Nevertheless, identification of time delays is no easy work, because models with time delays generally fall into the class of functional differential equations with infinite dimensions (Anguelova and Wennberga (2008), Bayrak and Tatlicioglu (2012) , Belkoura. (2010), Zheng and Richard (2015), Zheng et al. (2011)). The need for robust observer in the case of non-linear time-delay systems is still present since several methods using the delay identification rely on the existence of such an observer.

In the present work, we propose a robust observer inspired by the work in Ghanes et al. (2013) but for a more general case we consider a class of non-linear time-delay systems with bounded state and delay with the triangular matrix in the state equation depending nonlinearly of the input. The first contribution of this paper is the conception of an observer with guaranty practical stability (which is proven by using the proposed Lyapunov-Krasovskii functional), the observation error converges to a ball depending on the upper bound of the chosen variable delay.

The second main contribution leads to propose a way to solve the problem of singularity on the input signal. More precisely there is a condition of persistency on one of the variable present in the state equation which can led to a loss of observability on some of the state variables. The proposed solution is to switch to an estimator when the

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condition is not met to ensure a rapid convergence when the observability is regained.

The paper is organized as follows. The system description is presented in Section II. In Section III, the observer design and its convergence analysis for a class of time-delay nonlinear systems in triangular form are given. Following in Section IV, simulation results highlight the performances of the proposed observer and the problem of singularities on the input are solved. Finally, in Section VI, some concluding remarks are given.

2. SYSTEM DESCRIPTION

The class of the chosen system consists in a time-delay non-linear system in strictly triangular form:

$$\Sigma_{\tau(t)}: \begin{cases} \dot{x}(t) = A(u(t))x(t) + \Psi(x(t), x_{\tau(t)}, u(t), u_{\tau(t)}), \\ t \geq 0 \\ y(t) = Cx(t), \\ x(s) = \varphi(s), \quad \forall s \in [-\tau^*, 0] \end{cases} \tag{1}$$

where $x_{\tau(t)} = x(t - \tau(t))$ and $u_{\tau(t)} = u(t - \tau(t))$ are respectively the delayed state and input, $x(t) \in R^n$ is the state of the system, $u(t) \in R^m$ is the input, $y(t) \in R$ represents the output of the system and

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}, \quad x_{\tau(t)} = \begin{pmatrix} x_{1,\tau(t)} \\ \vdots \\ x_{n,\tau(t)} \end{pmatrix},$$

$$A(u(t)) = \begin{pmatrix} 0 & F(u(t)) \\ 0 & 0 \end{pmatrix}, \quad C = (1 \cdots 0),$$

where $x_{i,\tau(t)} = x_i(t - \tau(t))$, for $i = 1, ..., n$ and $F(u(t)) = diag(f_1(u(t)), ..., f_{n-1}(u(t)))$

$\tau(t)$ is a positive and real-value function representing the unknown, variable time delay, affecting both the state and the input of the system which admits τ^* as an upper bound, and $x(s) = \varphi(s), \forall s \in [-\tau^*, 0]$ is an unknown continuous bounded initial function.

The vector function $\Psi(x, x_{\tau(t)}, u, u_{\tau(t)})$ is given by

$$\Psi(x, x_{\tau(t)}, u, u_{\tau(t)}) = \begin{pmatrix} \Psi_1(x_1, x_{1,\tau(t)}, u, u_{\tau(t)}) \\ \Psi_2(x_1, x_{1,\tau(t)}, x_2, x_{2,\tau(t)}, u, u_{\tau(t)}) \\ \vdots \\ \Psi_n(x, x_{\tau(t)}, u, u_{\tau(t)}) \end{pmatrix}$$

where the nonlinearities $\Psi_i(x_1, x_{1,\tau(t)}, ..., x_i, x_{i,\tau(t)}, u, u_{\tau(t)})$ have a triangular structure with respect to $x_1, ..., x_i$ and $x_{1,\tau(t)}, ..., x_{i,\tau(t)}$, for $i = 1, ..., n$.
(A,C) is on observable canonical form and Ψ is triangular inferior with respect to x and x_τ therefore, the system $\Sigma_{\tau(t)}$ (1) is uniformly observable for any input and time-delayed input.

To complete the description of system $\Sigma_{\tau(t)}$ (1), the following assumptions are considered for a delay unknown and variable.

A1. The state and the input are considered bounded[1], that is $x(t) \in \chi \subset R^n$ (that is a compact subset of R^n) and $u(t) \in U \subset R^m$ (that is a subset of R^m).
A2. The function $\Psi(x, x_{\tau(t)}, u, u_{\tau(t)})$ is globally Lipschitz (on χ) w.r.t x, $x_{\tau(t)}$ and $u_{\tau(t)}$, uniformly w.r.t. u.
A3. The time-varying delay satisfies the following properties:
i) $\exists\ \tau^* > 0$, such that $\sup(\tau(t))_{t\geq 0} \leq \tau^*$.
ii) $\exists\ \beta > 0$, such that $1 - \dot{\tau}(t) \geq \beta$.

3. OBSERVER DESIGN

Consider system (1), then an observer for the class of systems of the form (1) is given by

$$O_{\tau^*}: \begin{cases} \dot{z}(t) = A(u(t))z(t) + \Psi(z(t), z_{\tau^*}, u(t), u_{\tau^*}) \\ \qquad\quad -S^{-1}C^T\{\hat{y}(t) - y(t)\} \\ \hat{y}(t) = Cz(t) \end{cases} \tag{2}$$

where S is symmetric, positive definite and verify the following equation:

$$-\rho S - A^T S - SA + C^T C = \dot{S}. \tag{3}$$

Proposition 1. For seek of simplicity, hereinafter we have chosen the arbitrarily fixed time-delay observer (2) equal to τ^*. We also chose to have the notation $A = A(u(t))$ to facilitate the reading.

Let us now define $e = z - x$ the observation error, whose dynamics is

$$\begin{aligned} \dot{e} = &\{A - S^{-1}C^T C\}e \\ &+\Psi(z, z_{\tau^*}, u, u_{\tau^*}) - \Psi(x, x_{\tau(t)}, u, u_{\tau(t)}) \end{aligned} \tag{4}$$

Theorem 2. Suppose that assumptions A1-A3 are fulfilled and $\|\varepsilon(s)\| < \delta_1$ for any bounded $\delta_1 > 0$ and $\forall\ s \in [-\tau^*, 0]$. Then, $\exists\ \rho_0 \geq 1$ such that the observation error dynamics (4) is δ_2-practically stable[2] for all $\rho \geq \rho_0$ and for some bounded $\delta_2 > 0$.

Proof. In order to invoke assumptions A1 and A2, the term $\{\Psi(z, z_{\tau^*}, u, u_{\tau^*}) - \Psi(x, x_{\tau(t)}, u, u_{\tau(t)})\}$ is rewritten as follows by adding and subtracting $\Psi(x, x_{\tau^*}, u, u_{\tau^*})$

$$\begin{aligned} \Psi(z, z_{\tau^*}, u, u_{\tau^*}) - \Psi(x, x_{\tau(t)}, u, u_{\tau(t)}) = &\ \Psi(z, z_{\tau^*}, u, u_{\tau^*}) \\ &-\Psi(x, x_{\tau^*}, u, u_{\tau^*}) \\ &+\bar{\Psi}(x, x_{\tau^*}, u, u_{\tau^*}, x_{\tau(t)}, u_{\tau(t)}) \end{aligned}$$

where

$$\begin{aligned} \bar{\Psi}(x, x_{\tau^*}, u, u_{\tau^*}, x_{\tau(t)}, u_{\tau(t)}) := &\ \Psi(x, x_{\tau^*}, u, u_{\tau^*}) \\ &- \Psi(x, x_{\tau(t)}, u, u_{\tau(t)}) \end{aligned} \tag{5}$$

characterizes the difference between the term that depends on the upper bound of the unknown delay and the term which depends on the unknown delay.

Define the Lyapunov-Krasovskii candidate functional

[1] The boundedness of the state excludes implicitly all initial conditions that generate unbounded state.
[2] Roughly speaking, practical stability means that the observation error converges exponentially to a ball B_r with radius $r > 0$.