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Robust trajectory tracking for a feedback linearizable nonlinear system

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Abstract: For a nonlinear system, affine in the control and in the disturbance, a generalized tracking problem is considered. The prescribed trajectory and prescribed discrete points should be tracked robustly with respect to a disturbance bounded in L_2. It is assumed that the system has the full relative degree both with respect to the control and to the disturbance. Once the system is feedback linearized, the previous results on a robust tracking for a linear system are applied. The tracking condition is formulated in terms of an original system. Illustrative examples are presented.

Keywords: nonlinear system, feedback linearization, tracking problem.

1. INTRODUCTION

The tracking problem for a nonlinear system is widely presented in the literature (see e.g. Slotine (1983); Bentash (1990); Lu (1996); Li and Krstić (1997); Kamalapurkara et al. (2015) and references therein). In many publications, an original nonlinear system is linearized by two main approaches. The first approach exploits the Taylor series expansion along a nominal trajectory (Shinar (1981); Vukić et al. (2003)). The second approach is a feedback linearization, pioneered by Isidori (1989). In this approach, by using under some assumptions a suitable transformation of state variables and by definition of an auxiliary control input, one can treat an original nonlinear control problem as a linear one, thus applying all the variety of linear control theory results. Various applications are presented e.g. in Bezick et al. (1995); Monahemi and Krstić (1996); Devaud and Siguerdidjane (2002); Chen et al. (2004) and other papers. It should be noted that most of publications on the feedback linearization deal with a one-sided optimization problems, while the problems with a disturbing counterpart were investigated less (let us mention Isidori and Astolfi (1992); Mei et al. (1999); Chen et al. (2004))

A generalized tracking problem for a linear system with an unknown disturbance on a finite control interval was formulated by Turetskij (1989); Tretyakov and Turetsky (1995). In this formulation, the cost functional is a Lebesgue-Stieltjes integral of a weighted squared discrepancy between an actual and a prescribed system motion. This integral is generated by a measure, consisting of discrete and continuous components. The discrete measure represents a desire of a control designer to guide the system close to prescribed discrete points, while the continuous component corresponds to the problem of tracking a given trajectory at some time intervals in the sense of L_2. A tracking algorithm in the sense of minimization of such a functional, robust with respect to an unknown disturbance, was proposed by the author in Turetskij (1989) and developed by him and his co-authors in Shinar et al. (2008) and Turetsky et al. (2014). The case of a pure discrete measure (the route realization problem) is considered in Turetsky (2016). The robust tracking strategy is constructed as the optimal strategy in an auxiliary linear-quadratic differential game (LQDG). Penalty coefficients for the control and the disturbance expenditure are small, i.e. a cheap-control approach is utilized. Various specific problems can be obtained from this generalized tracking problem, depending on the controlled system, cost functional structure or specific prescribed trajectory.

In the present paper, the generalized tracking problem is formulated for a nonlinear feedback linearizable system. Once the system is linearized, a robust tracking by the cheap-control LQDG strategy can be applied. The tracking conditions for a linear system (Turetsky (1999); Shinar et al. (2008); Turetsky et al. (2014)) are translated into respective conditions in terms of an original nonlinear system.

2. PROBLEM STATEMENT

Consider a SISO system

$$\dot{x} = f(x) + g_1(x)u + g_2(x)v, \;\; x(0) = x_0, \;\; t \in [0, t_f], \quad (1)$$

$$y = h(x), \quad (2)$$

where $x \in \mathbb{R}^n$ is the state vector, u and v are scalar control and disturbance, respectively; y is the scalar output, t_f is the prescribed final time instant; sufficiently smooth vector functions $f(x)$, $g_1(x)$, $g_2(x)$ and scalar function $h(x)$ of suitable dimensions are defined on a domain $D \subseteq \mathbb{R}^n$, containing the origin.

Let $t_i \in (t_0, t_f]$, $i = 1, \ldots, K$, and $(a_j, b_j) \subset [t_0, t_f]$, $j = 1, \ldots, L$, be prescribed time instants and non-intersecting intervals, such that at least one of the conditions $t_K = t_f$, $b_L = t_f$, is satisfied. Let $\tilde{y}(t)$ $t \in [t_0, t_f]$, be the function, continuous on each interval $[a_j, b_j]$, $j = 1, \ldots, L$. Let define the cost functional

$$J = G(x(\cdot)) = \sum_{i=1}^{K} [y(t_i) - \tilde{y}(t_i)]^2 +$$

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$$\sum_{j=1}^{L} \int_{a_j}^{b_j} [y(t) - \tilde{y}(t)]^2 \, dt. \tag{3}$$

Let $T = \bigcup_{j=1}^{L} (a_j, b_j)$ and $\zeta(t)$ be an indicator function of T:

$$\zeta(t) = \begin{cases} 1, \ t \in T \\ 0, \ t \notin T \end{cases} \tag{4}$$

Let $\chi([a,b])$ be the number of the moments $t_i \in [a,b]$. Define the measure

$$\mu([a,b]) = \int_a^b \zeta(t) dt + \chi([a,b]). \tag{5}$$

Then the functional (3) can be rewritten in a form of a Lebesgue-Stilties integral

$$G(x(\cdot)) = \int_{[0,t_f]} [y(t) - \tilde{y}(t)]^2 \, d\mu(t), \tag{6}$$

Robust Tracking Problem (RTP). For any $\zeta > 0$ and for a given $\nu > 0$, to construct a feedback strategy $u_{\zeta\nu}(t,y)$ such that

$$G(x_{\zeta\nu}(\cdot)) \le \zeta, \tag{7}$$

for any $v(t) \in L_2[0,t_f]$, satisfying

$$\int_0^{t_f} |v(t)|^2 dt < \nu, \tag{8}$$

where $x_{\zeta\nu}(t)$ is a solution of (1), generated by $u_{\zeta\nu}(t,y)$ and $v(t)$, $y_{\zeta\nu}(t) = h(x_{\zeta\nu}(t))$; $L_2[t_0,t_f]$ denotes the space of square-integrable functions $f(\cdot): \ [t_0,t_f] \to \mathbb{R}$.

3. SOLUTION

3.1 Feedback Linearization

In this paper, for the sake of simplicity, it is assumed that the system (1) has the full relative degree n both with respect to the control and to the disturbance, yielding that its exact feedback linearizability. Namely, define the coordinate transformation

$$z = T(x) = \begin{bmatrix} h(x) \\ L_f^1 h(x) \\ \vdots \\ L_f^{n-1} h(x) \end{bmatrix}, \tag{9}$$

where

$$L_f^0 h(x) = h(x), \quad L_f^k h(x) = \sum_{i=1}^{n} \frac{\partial L_f^{k-1} h(x)}{\partial x_i} f_i(x), \tag{10}$$

are the Lie derivatives (Yano (1957)) of $h(x)$ with respect to $f(x)$. By (9), the original system (1) becomes

$$\begin{array}{lcl} \dot{z}_1 & = & z_2, \\ \dot{z}_2 & = & z_3, \\ & \dots & \\ \dot{z}_{n-1} & = & z_n, \\ \dot{z}_n & = & a(x(t)) + b(x(t))u(t) + c(x(t))v(t), \end{array} \tag{11}$$

where

$$a(x) = L_f^n h(x), \tag{12}$$

$$b(x) = L_{g_1} L_f^{n-1} h(x) \ne 0, \tag{13}$$

$$c(x) = L_{g_2} L_f^{n-1} h(x) \ne 0. \tag{14}$$

The output (2) becomes

$$y = z_1. \tag{15}$$

Thus, by denoting

$$U(t) = a(x(t)) + b(x(t))u(t), \tag{16}$$

$$V = c(x(t))v(t), \tag{17}$$

the system (1) becomes

$$\dot{z} = Az + BU + BV, \ z(0) = T(x_0), \quad t \in [0,t_f], \tag{18}$$

where

$$A = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 \\ & & & \vdots & & \\ 0 & 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}. \tag{19}$$

3.2 Auxiliary LQDG

Consider an auxiliary LQDG for (18) with the cost functional

$$J_{\alpha\beta} = J_{\alpha\beta}(U(\cdot), V(\cdot)) = \int_{[0,t_f]} [z_1(t) - \tilde{y}(t)]^2 \, d\mu(t) +$$

$$\alpha \int_0^{t_f} U^2(t) dt - \beta \int_0^{t_f} V^2(t) dt, \tag{20}$$

to be minimized by U and maximized by V. The function $\tilde{y}(t)$ and the measure $d\mu(t)$ in (20) are the same as in (6); $\alpha, \beta > 0$ are control and disturbance penalty coefficients. Due to (2) and (9), the first term of (20) coincides with $G(x(\cdot))$.

If the penalty coefficients of (20) satisfy

$$\alpha \le \beta, \tag{21}$$

then, due to Shinar et al. (2008), the LQDG (18), (20) is solvable and the minimizer's optimal strategy is

$$U^0_{\alpha\beta}(t,z) = -\frac{1}{2\alpha} B^T l^0_{\alpha\beta}(t,z), \tag{22}$$

where

$$l^0_{\alpha\beta}(t,z) = \Phi^T(t_f,t) \Big(2R_{\alpha\beta}(t) \Phi(t_f,t) z + r_{\alpha\beta}(t) \Big), \tag{23}$$

$\Phi(t,\tau)$ is the fundamental matrix of the homogenous equation $\dot{z} = Az$. The matrix function $R_{\alpha\beta}(t)$ and the vector function $r_{\alpha\beta}(t)$ satisfy the impulsive differential equations

$$\frac{dR}{dt} = -RQ_{\alpha\beta}(t)R - \zeta(t)S(t). \tag{24}$$

$$R(t_f + 0) = 0, \quad R(t_i + 0) - R(t_i) = -S(t_i), \tag{25}$$

$$\frac{dr}{dt} = -R_{\alpha\beta}(t) Q_{\alpha\beta}(t) r + 2\zeta(t)\tilde{y}(t) X^T(t,t_f) D^T, \tag{26}$$

$$r(t_f + 0) = 0, \ \ r(t_i + 0) - r(t_i) = 2\tilde{y}(t_i) \Phi^T(t_i,t_f) D^T, \tag{27}$$

where $i = 1, \dots, K$, $\zeta(t)$ is defined by (4),

$$Q_{\alpha\beta}(t) = \left(\frac{1}{\beta} - \frac{1}{\alpha} \right) \Phi(t_f,t) BB^T \Phi^T(t_f,t), \tag{28}$$

$$S = \Phi^T(t,t_f) D^T D \Phi(t,t_f), \tag{29}$$

$$D = [1, 0, \dots, 0]. \tag{30}$$