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IFAC-PapersOnLine 49-18 (2016) 588–593

Path Following for Underactuated Marine Vessels

D.J.W. Belleter * **C. Paliotta** * **M. Maggiore** ** **K.Y. Pettersen** *

* *Centre for Autonomous Marine Operations and Systems (NTNU AMOS) and the Department of Engineering Cybernetics, Norwegian University of Science and Technology, NO7491 Trondheim, Norway*
{dennis.belleter,claudio.paliotta,kristin.y.pettersen}@itk.ntnu.no
** *Department of Electrical and Computer Engineering, University of Toronto, 10 King's College rd., Toronto, ON, M5S3G4, Canada,*
maggiore@ece.utoronto.ca

Abstract: This paper investigates the problem of making an underactuated marine vessel follow an arbitrary differentiable Jordan curve. A solution is proposed which relies on a hierarchical control methodology involving the simultaneous stabilization of two nested sets, and results in a smooth, static, and time-invariant feedback. The methodology in question effectively reduces the control problem to one of path following for a kinematic point-mass. It is shown that as long as the curvature of the path is smaller than a quantity dependent on the mass and damping parameters of the ship, path following is achieved with uniformly bounded sway speed.

1. INTRODUCTION

This paper presents a control methodology for underactuated marine vessels with two control inputs (thrust and torque) and three degrees-of-freedom (position and rotation). The control specification is path following: make the ship approach a path and follow it with nonzero speed without requiring any time parametrization. While in the trajectory tracking problem one would seek to make the ship follow a moving reference point, in path following one wants to stabilize a suitable controlled-invariant subset of the state space (see Nielsen et al. (2010)), and no exogenous signal drives the control loop.

The path following and trajectory tracking problems have been the subject of significant research in the context of marine vessels. We mention some of the relevant references. Straight-line/waypoint path following for underactuated vessels is considered in Fredriksen and Pettersen (2006), Børhaug et al. (2008), Oh and Sun (2010), and Aguiar and Pascoal (2007). Path following for curved paths is considered in Do and Pan (2006) where the path is parametrized by a path-variable that propagates along the path with a velocity dependent on the desired vessel velocity. The papers Aguiar and Hespanha (2007) and Skjetne et al. (2005) investigate the trajectory tracking problem. Path following of curved paths for underactuated vessels using a Serret-Frenet path frame is considered in Li et al. (2009); Moe et al. (2014) and Lapierre and Soetanto (2007).

The papers listed above consider path following of straight-line paths or path-following/trajectory-tracking of curved paths that are parametrized by time or a path variable. To the best of our knowledge, in the context of marine vessels, the problem of finding a smooth, static, and time invariant feedback solving the path following problem for general unparametrized paths remains open. In this paper, we make an initial step towards its solution. Our approach leverages the hierarchical control methodology presented in El-Hawwary and Maggiore (2013), a methodology which has been used in Roza and Maggiore (2014) to derive almost global position controllers for underactuated flying vehicles. The idea is to first design a path following control law for a kinematic point-mass. Then from this feedback extract a desired heading angle, and view it as a reference for a torque controller. Carrying out these two separate design steps corresponds to the simultaneous stabilization of two nested subsets of the state space, and the a reduction theorem from El-Hawwary and Maggiore (2013) is used to show overall stability. In particular, we show that if the curvature of the path is not too large in relation to a constant that depends on the ship's parameters, then the sideways velocity is uniformly bounded.

The challenge in solving the path following problem for marine vessels is that, due to the presence of sideways motion, in order to stay on a curved path the ship cannot head tangent to it, and its angle of attack relative to the path's tangent depends on the sway speed.

2. PRELIMINARIES AND NOTATION

In this paper we adopt the following notation. We denote by $\mathbb{S}^1$ the set of real numbers modulo 2π, with the differentiable manifold structure making it diffeomorphic to the unit circle. If $\psi \in \mathbb{S}^1$, R_ψ is the rotation matrix

[1] M. Maggiore was supported by the Natural Sciences and Engineering Research Council of Canada (NSERC). This research was performed while M. Maggiore was on sabbatical leave at the Laboratoire des Signaux et Systèmes, Gif sur Yvette, France.
[2] D.J.W. Belleter, C. Paliotta, and K.Y. Pettersen were supported by the Research Council of Norway through its Centres of Excellence funding scheme, project No. 223254 AMOS.

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2016.10.229

$$R_\psi = \begin{bmatrix} \cos(\psi) & -\sin(\psi) \\ \sin(\psi) & \cos(\psi) \end{bmatrix}.$$

If $f(x, y)$ is a differentiable function of two scalar variables, we denote by $\partial_x f$, $\partial_y f$ the partial derivatives with respect to x and y, respectively. Similarly, we define $\partial^2_{xy} f := \partial_x \partial_y f$, and similarly for the other second-order partial derivatives. If $f : \mathbb{R}^n \to \mathbb{R}^m$ is a differentiable vector function and $p \in \mathbb{R}^n$, df_p is the $m \times n$ Jacobian matrix of f at p. If Γ is a closed subset of a metric space (M, d) and $x \in M$, then we denote by $\|x\|_M$ the point-to-set distance of x to M, $\|x\|_M = \inf_{y \in M} d(x - y)$.

The following stability definitions are taken from El-Hawwary and Maggiore (2013). Let $\Sigma : \dot{\chi} = f(\chi)$ be a smooth dynamical system with state space a Riemannian manifold $\mathcal{X}$ with associated metric d. Let $\phi(t, \chi_0)$ denote the local phase flow generated by Σ, and let $B_\delta(x)$ denote the ball of radius δ centred at $x \in M$.

Consider a closed set $\Gamma \subset \mathcal{X}$ which is positively invariant for Σ, i.e., for all $\chi_0 \in \Gamma$, $\phi(t, \chi_0) \in \Gamma$ for all $t > 0$ for which $\phi(t, \chi_0)$ is defined. Then we have the following stability definitions taken from El-Hawwary and Maggiore (2013).

Definition 1. The set Γ is *stable* for Σ if for any $\varepsilon > 0$, there exists a neighborhood $\mathcal{N}(\Gamma) \subset \mathcal{X}$ such that, for all $\chi_0 \in \mathcal{N}(\Gamma)$, $\phi(t, \chi_0) \in B_\varepsilon(\Gamma)$, for all $t > 0$ for which $\phi(t, \chi_0)$ is defined. The set Γ is *attractive* for Σ if there exists a neighborhood $\mathcal{N}(\Gamma) \subset \mathcal{X}$ such that for all $\chi_0 \in \mathcal{N}(\Gamma)$, $\lim_{t\to\infty} \|\phi(t, \chi_0)\|_\Gamma = 0$. The *domain of attraction of* Γ is the set $\{\chi_0 \in \mathcal{X} : \lim_{t\to\infty} \|\phi(t, \chi_0)\|_\Gamma = 0\}$. The set Γ is *globally attractive* for Σ if it is attractive with domain of attraction $\mathcal{X}$. The set Γ is *locally asymptotically stable (LAS)* for Σ if it is stable and attractive. The set Γ is *globally asymptotically stable* for Σ if it is stable and globally attractive. If $\Gamma_1 \subset \Gamma_2$ are two closed positively invariant sets, then Γ_1 is *asymptotically stable relative to* Γ_2 if Γ_1 is asymptotically stable for the restriction of Σ to Γ_2. System Σ is *locally uniformly bounded (LUB) near* Γ if for each $x \in \Gamma$ there exist positive scalars λ and m such that $\phi(\mathbb{R}_+, B_\lambda(x)) \subset B_m(x)$. △

The following result is key in the development of this paper.

Theorem 1. (El-Hawwary and Maggiore (2013)). Let Γ_1, Γ_2, $\Gamma_1 \subset \Gamma_2 \subset \mathcal{X}$, be two closed sets that are positively invariant for Σ and suppose that Γ_1 is not compact. If

(i) Γ_1 is asymptotically stable relative to Γ_2,
(ii) Γ_2 is asymptotically stable, and
(iii) Σ is LUB near Γ_1,

then Γ_1 is asymptotically stable for Σ.

3. THE PROBLEM

Consider the 3-degrees-of-freedom vessel depicted in Figure 1, which may describe an autonomous surface vessel (ASV) or an autonomous underwater vehicle (AUV) moving in the horizontal plane. We denote by $p \in \mathbb{R}^2$ the position of the vessel on the plane and $\psi \in \mathbb{S}^1$ its heading (or yaw) angle. The yaw rate $\dot{\psi}$ is denoted by r.

We attach at the point p of the vessel a body frame aligned with the main axes of the vessel, as depicted in the figure, with the standard convention that the z-axis points into the plane (towards the sea bottom). We represent the velocity vector $\dot{p}$ in body frame coordinates as (u, v), where u, the longitudinal component of the velocity vector, is called the surge speed, while v, the lateral component, is called the sway speed. Finally, the control inputs of the vessel are the surge trust T_u and the rudder angle T_r. In terms of these variables, the model derived in Fossen (2011) is

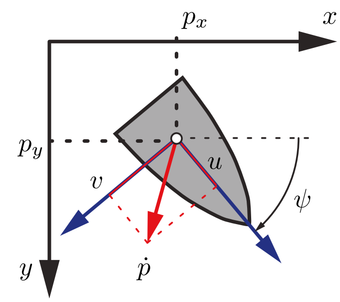

Fig. 1. Illustration of the ship's kinematic variables.

$$\begin{aligned} \dot{\boldsymbol{\eta}} &= \begin{bmatrix} R_\psi & 0 \\ 0 & 1 \end{bmatrix} \boldsymbol{\nu} \\ \boldsymbol{M}\dot{\boldsymbol{\nu}} + \boldsymbol{C}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{D}\boldsymbol{\nu} &= \boldsymbol{B}\boldsymbol{f} \end{aligned} \tag{1}$$

with $\boldsymbol{\eta} \triangleq [p, \psi]^\top$, $\boldsymbol{\nu} \triangleq [u, v, r]^\top$, and $\boldsymbol{f} \triangleq [T_u, T_r]^\top$. The matrices $\boldsymbol{M}$, $\boldsymbol{D}$, and $\boldsymbol{B}$ are given by

$$\boldsymbol{M} \triangleq \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & m_{23} \\ 0 & m_{23} & m_{33} \end{bmatrix}, \quad \boldsymbol{D} \triangleq \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & d_{23} \\ 0 & d_{32} & d_{33} \end{bmatrix}, \boldsymbol{B} \triangleq \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \\ 0 & b_{32} \end{bmatrix}$$

with $\boldsymbol{M} = \boldsymbol{M}^\top > 0$ the symmetric positive definite inertia matrix including added mass, $\boldsymbol{D} > 0$ is the hydrodynamic damping matrix, and $\boldsymbol{B}$ is the actuator configuration matrix. The matrix $\boldsymbol{C}(\boldsymbol{\nu})$ is the matrix of Coriolis and centripetal forces and can be obtained from $\boldsymbol{M}$ (see Fossen (2011)). We place the origin of the body frame at a point on the center-line of the vessel with distance ϵ from the centre of mass. Following Fredriksen and Pettersen (2006), assuming that the vessel is starboard symmetric, there exists ϵ such that the resulting dynamics have mass and damping matrices satisfying this relation: $\boldsymbol{M}^{-1}\boldsymbol{B}\boldsymbol{f} = [\tau_u, 0, \tau_r]^\top$. Thus, with this choice of origin of the body frame, the sway dynamics become decoupled from the rudder control input, making it easier to analyze the stability properties of the sway dynamics. Using this convention, the model of the marine vessel (1) can be represented as

$$\begin{aligned} \dot{p} &= R_\psi \begin{bmatrix} u \\ v \end{bmatrix} \\ \begin{bmatrix} \dot{u} \\ \dot{v} \end{bmatrix} &= \begin{bmatrix} F_u(v, r) - \frac{d_{11}}{m_{11}} u + \tau_u \\ X(u) r + Y(u) v \end{bmatrix} \\ \dot{\psi} &= r \\ \dot{r} &= F_r(u, v, r) + \tau_r. \end{aligned} \tag{2}$$

The functions $X(u)$ and $Y(u)$ are linear. Their expressions are given in Appendix A together with those of F_u and F_r. Denoting by $\chi := (p, u, v, \psi, r)$ the state of the vessel, the state space is $\mathcal{X} := \mathbb{R}^2 \times \mathbb{R} \times \mathbb{R} \times \mathbb{S}^1 \times \mathbb{R}$.

Assumption 1. We assume that $Y(u) < 0 \; \forall u \in [0, V_{\max}]$.

This is a realistic assumption, since $Y(\bar{u}) \geq 0$ would imply that the sway dynamics are undamped or unstable when the yaw rate r is zero.