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On the orthogonal subgrid scale pressure stabilization of finite deformation J2 plasticity

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Abstract

The use of stabilization methods is becoming an increasingly well-accepted technique due to their success in dealing with numerous numerical pathologies that arise in a variety of applications in computational mechanics.

In this paper a multiscale finite element method technique to deal with pressure stabilization of nearly incompressibility problems in nonlinear solid mechanics at finite deformations is presented. A J2-flow theory plasticity model at finite deformations is considered. A mixed formulation involving pressure and displacement fields is used as starting point. Within the finite element discretization setting, continuous linear interpolation for both fields is considered. To overcome the Babuška–Brezzi stability condition, a multiscale stabilization method based on the orthogonal subgrid scale (OSGS) technique is introduced. A suitable nonlinear expression of the stabilization parameter is proposed. The main advantage of the method is the possibility of using linear triangular or tetrahedral finite elements, which are easy to generate and, therefore, very convenient for practical industrial applications.

Numerical results obtained using the OSGS stabilization technique are compared with results provided by the P1 standard Galerkin displacements linear triangular/tetrahedral element, P1/P1 standard mixed linear displacements/linear pressure triangular/tetrahedral element and Q1/P0 mixed bilinear/trilinear displacements/constant pressure quadrilateral/hexahedral element for 2D/3D nearly incompressible problems in the context of a nonlinear finite deformation J2 plasticity model.

Keywords: Multiscale methods; Subgrid scale methods; Orthogonal subgrid scale methods; Stabilized finite element methods; Stabilization; Incompressibility; Plasticity; Finite deformation

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1. Introduction

The use of stabilized methods is becoming an increasingly well-accepted technique due to their success in dealing with numerous numerical pathologies that arise in a variety of applications in computational mechanics. This paper deals with the application of multiscale methods, in particular the orthogonal subgrid scale (OSGS) method, to the pressure stabilization of nearly incompressibility problems in nonlinear solid mechanics at finite deformations using low order finite elements. A Finite deformation J2 plasticity model is considered. The goal is to consistently derive, within the framework of the OSGS method, a modified variational mixed formulation of the original problem with enhanced stability properties.

It is well known that the standard irreductible Galerkin finite element method with low-order piecewise polynomials perform miserably in nearly incompressible problems, exhibiting spurious wild oscillations of the mean pressure and leading to a response which is almost completely locked due to the incompressibility constraint. In the computational literature these devastating numerical difficulties are referred to as *locking* phenomena. Actually, the exact incompressibility problem does not admit an irreductible formulation and, consequently, a mixed displacement/pressure framework is necessary in that case. Even though, many standard mixed finite element formulations, particularly those using low order interpolations, also perform poorly or totally fail to perform for nearly incompressibility or incompressibility problems, producing results throughly polluted by spurious oscillations of the pressure.

To overcome these difficulties, over the years different strategies were suggested to reduce or avoid volumetric locking and pressure oscillations in finite element solutions. For an engineering oriented presentation see the well known books of Zienkiewicz and Taylor [45], Hughes [19] and Simo and Hughes (1998) [39]. For a more mathematically oriented presentation see the book of Brezzi and Fortin [3]. Different mixed and enhanced finite element formulations were proposed and degrees of success were obtained. See, e.g., R.L. Taylor [43], Simo et al. [42], Simo [35,36,40], Miehe [28], Simo and Rifai [41] and Simo and Armero [37]. Unfortunately, few approaches were successfully applied to low order finite elements, as shown for instance in Reddy and Simo [32] for the enhanced assumed strain method or R.L. Taylor [43] for the mixed method. This was due to the strictness of the inf–sup or Ladyzhenskaya–Babuška–Brezzi (LBB) condition when the standard Galerkin finite element projection was straightforwardly applied to mixed low order finite elements, as it imposes severe restrictions on the compatibility of the interpolations used for the displacement and pressure fields [3,45]. One significant effort in that direction was the so called mini element [1], an attractive linear displacement/pressure triangle enhanced with a cubic displacement bubble function. The mini-element satisfies the LBB condition, but it is only marginally stable and it does not perform very well in many practical situations. Despite these not very good satisfactory results, there still exists a great practical interest in the use of stable low order elements, mainly motivated by the fact that, nowadays, tetrahedral finite element meshes are relatively easy to generate for real life complex geometries. Therefore, stabilization techniques for low order finite elements is a very active research area in solid mechanics. Some recent formulations have been proposed by Zienkiewicz et al. [46], Klaas et al. [24], Oñate et al. [29,30], Maniatty et al. [26,27], Maniatty and Liu [25], Reese and Wriggers [33], Reese et al. [34], and Ramesh and Maniatty [31]. In Zienkiewicz et al. [46] a stabilization term, arising from a fractional step method, is introduced using a mixed displacement/pressure formulation and linear triangles and tetrahedra, within the framework of explicit dynamic codes for solids. In Klaas et al. [24] a stabilized formulation for large deformation elasticity using P1/P1 elements was presented, where stabilization was attained by adding a mesh-dependent stabilization term, which can be viewed as a perturbation of the test functions, leading to a Galerkin least square (GLS) stabilized discrete weak form. This formulation has been recently extended to elastoplastic finite deformation problems by Ramesh and Maniatty [31]. An extension to higher order interpolation functions, using a local reconstruction method to construct part of the stabilization terms, was presented by Maniatty et al. [27]. Maniatty et al. [26] also presented a stabilized formulation for steady state flow problems to simulate forming problems such as drawing. In Oñate