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Circle Condition-Based Robust Feedback Control Against Plant Perturbation

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Abstract: This paper presents a novel robust feedback (FB) controller design against plant perturbation for the fast and precise motion control of high-precision servo systems. The plant perturbation, e.g., parameter fluctuation and uncertainties, due to temperature variation and/or aged deterioration generally deteriorates the motion performance as well as the system stability. Balancing a trade-off between expansion of the control bandwidth and the stability, therefore, is a general and important index in the FB controller design to provide robust properties against the perturbation. In this study, a circle condition-based robust FB controller design considering the robust stability and the robust sensitivity is proposed to achieve the improved motion control performance. Effectiveness of the proposed approach has been verified by numerical simulations for a galvano scanner.

Keywords: Circle condition, feedback control, robustness, plant perturbation, linear matrix inequality (LMI).

1. INTRODUCTION

The fast-response and high-precision motion control is one of indispensable techniques in a wide variety of high performance industrial mechatronic systems such as data storage devices, machine tools, and industrial robots, from viewpoints of high productivity, high quality of products, and reduction of energy consumption (see Iwasaki et al. (2012) as a notable example). However, it is well known that parameter fluctuation, e.g., torque constant of actuator, resonance frequency, and moment of inertia, and uncertainties in electrical and mechanical components ("plant perturbation" in the following) deteriorate the motion control performance as well as the system stability. Hence, an effective controller design which considers robustness against the perturbation is one of the important issues to be solved in this research field.

In order to overcome the issue above, a two-degree-of-freedom (2DoF) control framework using feedforward (FF) control and feedback (FB) control is one of practical and promising techniques. In the 2DoF control design, the FB control is generally aimed at suppressing effects of the plant perturbation and/or disturbances, and an appropriate disturbance suppression characteristic, e.g., expansion of the control bandwidth and sensitivity shaping at the specified frequencies, should be designed with the desired system stability satisfied. Design of the desired disturbance suppression characteristic, however, is difficult to be achieved in the case that the plant perturbation exceedingly affects the stability.

A variety of robust FB control techniques have been discussed in the former literature, e.g., the H_∞ control (see Nie et al. (2013)), the linear matrix inequality (LMI)-based FB control (see Iwasaki et al. (2005)), the optimal FB controller tuning schemes (see Low et al. (2007)), the phase stabilization of resonant modes (see Atsumi et al. (2007)), the state FB control (see Morais et al. (2013)), etc. The authors have also proposed a circle condition-based FB controller design, using the LMI-based optimization technique (see Maeda et al. (2014)). The circle condition imposes a constraint on the stability margins, i.e., gain margin and phase margin, while the pole placement determines the control bandwidth of the FB control system. The controller design can pursue the disturbance suppression performance while satisfying the specified stability. However, since the plant perturbation has not yet been considered in the design, improvement of the robustness still remains as a future work.

In this paper, a novel circle condition-based robust FB controller design is presented to provide the fast and precise motion control of high-precision servo systems with the plant perturbation. The proposed approach basically stands on the conventional approach (namely, LMI-based optimization technique), and newly introduces circle conditions for the robust stability and the robust sensitivity, by regarding the plant perturbation as structured (i.e., "known" parameter fluctuations and modeling errors) and unstructured (i.e., "unknown" parameter fluctuations and modeling errors) uncertainties in the circle condition design. Effectiveness of the proposed approach has been verified by numerical simulations using a galvano scanner, with comparison to the conventional approach.

2. CIRCLE CONDITION-BASED ROBUST FEEDBACK CONTROLLER DESIGN

In this section, at first, a basic FB controller design using the circle condition is briefly explained as the conventional approach (see Maeda et al. (2014)). Then, it is extended

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to a new circle condition design considering the robust properties against the plant perturbation.

2.1 Basic Design

Fig. 1 shows a block diagram of a general cascade-type FB control system, where $P(z) = N(z)/D(z)$ is the plant, $C_{cc}(z)$ is the circle condition-based FB controller, $y(z)$ is the system output, $r(z)$ is the reference signal, $d(z)$ is the disturbance, and $u_{fb}(z)$ is the FB control input, respectively. $C_{cc}(z)$ is mathematically defined as follows:

$$C_{cc}(z) = -\frac{1}{P_d(z)}\frac{N_{cc}(z)}{N_{cc}(z)+D_{cc}(z)}, \tag{1}$$
$$N_{cc}(z) = a_n z^n + a_{n-1}z^{n-1} + \cdots + a_1 z + a_0 = \boldsymbol{z}\boldsymbol{a},$$
$$\boldsymbol{a} = [\, a_n \ a_{n-1} \ \cdots \ a_1 \ a_0 \,]^T,$$

where $P_d(z)$ is the design plant model with the relative order of m, $D_{cc}(z)$ is the stable polynomial with the order of n_{dcc} to determine the response poles, and the undetermined coefficient vector $\boldsymbol{a} \in \mathcal{R}^{n+1}$ is a design parameter with the order of n ($n \le n_{dcc} - m$).

At first, in order to converge the output $y(z)$ for the disturbance $d(z)$ by the specified poles, the design of $\boldsymbol{a}$ should satisfy the following constraints:

i) Polynomial $N_{cc}(z)+D_{cc}(z)$ includes all roots λ_q ($q = 1, 2, \ldots, n_p$) of the denominator $D(z)$ in $P(z)$.
ii) Steady characteristic of $y(z)$ for $d(z)$ becomes zero.

The constraints i) and ii) can be formulated by the following matrix equation as an affine form of $\boldsymbol{a}$:

$$\boldsymbol{\Sigma}\boldsymbol{a} = \boldsymbol{\Gamma}, \tag{2}$$

where $\boldsymbol{\Sigma} \in \mathcal{C}^{(n_p+1)\times(n+1)}$ and $\boldsymbol{\Gamma} \in \mathcal{C}^{(n_p+1)\times 1}$.

Next, a circle condition for stability margins, i.e., gain margin and phase margin, is defined as an inequality constraint. Fig. 2 shows an example of Nyquist diagram of the open-loop characteristic $L(z) = C_{cc}(z)P(z)$. In the figure, C_{sm} is the specified circle with the center of $(-\sigma_{sm}, j0)$ and the radius of r_{sm}. The distance between the Nyquist point $(-1, j0)$ and the intersection of C_{sm} on the real axis corresponds to the gain margin G_m ($g_m = 20\log_{10}G_m$ [dB]), while the angle between two intersections of C_{sm} and the unit circle corresponds to the phase margin Φ_m ($\phi_m = 180\Phi_m/\pi$ [deg]). A condition such that the Nyquist trajectory of $L(z)$ does not pass through the inside of C_{sm} at a frequency ω can be mathematically formulated as follows:

$$|L(j\omega) + \sigma_{sm}| > r_{sm}, \tag{3}$$
$$\sigma_{sm} = \frac{G_m^2 - 1}{2G_m(G_m\cos\Phi_m - 1)},$$
$$r_{sm} = \frac{(G_m - 1)^2 + 2G_m(1 - \cos\Phi_m)}{2G_m(G_m\cos\Phi_m - 1)}.$$

In order to stabilize the FB control system, C_{sm} should contain $(-1,\ j0)$ inside, i.e., $0 < r_{sm} < \sigma_{sm}$, and not contain the origin O inside, i.e., $(\sigma_{sm} - 1)^2 < r_{sm}^2$. By substituting $C_{cc}(z)$ of (1) to (3), the following inequality constraint can be formulated as a quadratic function of $\boldsymbol{a}$:

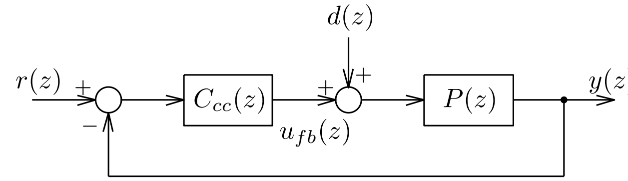

Fig. 1. Cascade-type FB control system with disturbance.

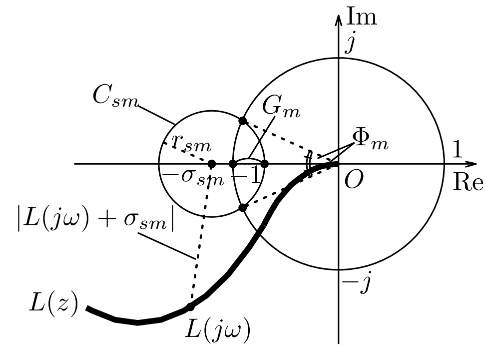

Fig. 2. Circle condition on Nyquist diagram.

$$\boldsymbol{a}^T\boldsymbol{Q}_{sm2}(\omega)\boldsymbol{a} + 2\boldsymbol{Q}_{sm1}(\omega)\boldsymbol{a} + Q_{sm0}(\omega) > 0, \tag{4}$$

where $\boldsymbol{Q}_{sm2}(\omega) \in \mathcal{R}^{(n+1)\times(n+1)}$, $\boldsymbol{Q}_{sm1}(\omega) \in \mathcal{R}^{1\times(n+1)}$, and $Q_{sm0}(\omega) \in \mathcal{R}^{1\times 1}$.

Finally, the parameter $\boldsymbol{a}$ which has the desired control bandwidth and the specified stability margins can be optimally designed, by using the following evaluation function J under the constraints of (2) and (4) defined by an LMI technique:

$$J = \boldsymbol{a}^T\boldsymbol{Q}_j\boldsymbol{a}, \tag{5}$$

where $\boldsymbol{Q}_j = \boldsymbol{I} \in \mathcal{R}^{(n+1)\times(n+1)}$ is the weighting matrix to minimize jerk components in $u_{fb}(z)$.

2.2 Circle Condition for Robust Stability and Robust Sensitivity

In order to design circle conditions for the robust stability and the robust sensitivity against the plant perturbation, a perturbed plant $\mathcal{P}$ is defined by (6) as a set of structured/unstructured plants $P_k(z) (k = 1, 2, \ldots, q)$.

$$\mathcal{P} = \{P_k(z) \mid P_k(z) = P_{sk}(z)\Delta_{us}(z)\}, \tag{6}$$

where $P_{sk}(z)$ is the plant characteristic with a structured uncertainty $\Delta_{sk}(z)$ for known parameter fluctuation and known modeling errors, and is mathematically defined as

$$P_{sk}(z) = P_n(z)\Delta_{sk}(z). \tag{7}$$

$\Delta_{us}(z)$ in (6), on the other hand, is the unstructured uncertainty for unknown perturbation and unknown modeling errors. By introducing a weighting function $W_{us}(z)$ which specifies the upper boundary of $|\Delta_{us}(z)|$, the mathematical formulation of $\Delta_{us}(z)$ can be expressed by