Available online at www.sciencedirect.com

ScienceDirect

IFAC PapersOnline CONFERENCE PAPER ARCHIVE

IFAC-PapersOnLine 49-21 (2016) 234–239

Optimal Actuator Design for Optimal Output Controllability

Philipp Wittmuess [*] **Michael Heidingsfeld** [*] **Oliver Sawodny** [*]

[*] *Institute for System Dynamics, University of Stuttgart, Waldburgstrasse 17/19, 70569 Stuttgart, Germany (e-mail: {philipp.wittmuess|michael.heidingsfeld|oliver.sawodny}@isys.uni-stuttgart.de)*

Abstract: For systems with a great design flexibility regarding the choice of inputs, by designing the actuators themselves, finding a good set of inputs or a good actuator layout, is not trivial. In this contribution, a measure for the correlation and redundancy of inputs to the system based on an Gramian-like output controllability matrix is presented. The input space is orthogonalized with respect to this measure. The main purpose of this input measure is to design new actuators for the system. It can also be used to identify redundant inputs that can be removed. The presented method yields sets of appropriate actuators for the system that give good controllability of the outputs. The method can be applied to large scale linear systems that result from the investigation of FE-models. In this scope, it yields for example the layout of optimal force inputs.
The validity of the approach is demonstrated by an illustrative example.

1. INTRODUCTION

While most methods in automatic control focus on the design of good feedback laws that control and stabilize dynamical systems, relatively little attention has been drawn to the systematic design of actuators that can be used to efficiently control a system. Also, it may be up to the control engineer to place the available actuators appropriately (Brown et al. (1999)).
The optimal actuator design problem is highly linked to the actuator-placement problem because the quality criteria for a good set of actuators and a good actuator layout are very similar. The actuator placement problem and the dual sensor placement problem have been studied mainly for huge mechanical systems (Lim (1992); Han and Lee (1999); Weickgenannt et al. (2011); Brown et al. (1999)). The actuator design problem mainly addresses distributed force actuators. Such actuators are used for example in continuum robot manipulators (Walker et al. (2005)). However, the presented method is not restricted to mechanical systems.

In the present contribution, we define a quality criterion based on the output controllability and show how it can be used for designing an optimal input. The criterion is based on the redundancy of outputs. The redundancy is measured by the scalar product of the output signals that can be generated using the two inputs.

It is shown by the analysis that the proposed method can also be used to identify sets of redundant actuators. Hence, in addition to revealing optimal actuator layouts, it gives an indication on how many actuators are actually required.

This paper is organized as follows: The optimal actuator design problem is formalized in section II. A measure for the correlation or redundancy of inputs is developed in section 3. Section 4 solves the optimal actuator design problem using the previously defined redundancy measure. The method is applied to an illustrative example in section 5. The paper closes with an brief conclusion and an outlook in section 6.

2. PROBLEM SETTING

2.1 System under investigation

Assume an asymptotically stable linear MIMO-system with n states, m' outputs and an input space of dimension m. The elements of the input space $b_i \in \mathbb{C}^n$ with $i = 1,\dots,m$ can be considered a representation of the design space for the actuators. All possible actuator configurations must be given by linear combinations of one or several b_i. The elements of the input space are collected in the input matrix $B = [b_1,\dots,b_m]$ with $m \in \mathbb{N}$ being the total number of possible inputs. Be $b_1,\dots,b_m \in \mathbb{C}^n$ and hence $B \in \mathbb{C}^{n\times m}$. Let the system matrix $A \in \mathbb{C}^{n\times n}$ and output matrix $\mathbb{C}^{m'\times n}$ be given. The inputs and outputs must be normalized to give appropriate results. Now, the system with state $x(t) \in \mathbb{C}^n$, input $u(t) \in \mathbb{C}^m$ from the space of all possible inputs and output $y \in \mathbb{C}^{m'}$ can be written

$$G: \quad \begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

The objective is now to find a projection matrix $V \in \mathbb{C}^{m\times\hat{m}}$ which implicitly defines a new input $\hat{u}$ for the system by the projection $u(t) = V\hat{u}(t)$. It is, of course not possible to get better controllability for $\hat{m} \le m$ but it may well be possible to omit a certain number of inputs without a significant loss of controllability or to combine a number of

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2016.10.556

inputs. The actuator layouts are then given by the columns of $V = [v_1, \ldots, v_m]$. Each v_i corresponds to an actuator which brings an input load Bv_i onto the system.

2.2 Objective

Depending on the problem under investigation, several design objectives for the projection V may arise:

(1) If one wants to use as few inputs as possible, $\hat{m}$ should be as low as possible.
(2) There may be redundant input configurations that shall be identified and removed to reduce the number of inputs.
(3) There may be actuator configurations to G that have no effect on the output at all since they only drive the zero dynamics. Such actuators can be neglected in control design.
(4) If one wants to design a novel distributed actuator, one may admit any linear combination of inputs. This problem arises especially if one has a finite-element-model and one wants to design a distributed actuator.

3. INPUT CORRELATION MEASURE

In this section, a scalar product on the input space is presented. It defines the correlation of two inputs with respect to the measurable difference at the outputs. The norm induced by the scalar product is a measure for output energy that can be generated using a specific input.

The basic idea to the construction of the scalar product is to use the observability Gramian for calculating the scalar product of two output signals. The output signals are defined by initial system states at time $t = 0$ which are given by a Dirac input. In the following, $(\circ)^H$ denotes the complex conjugate transpose of a matrix.

3.1 Observability Gramian

The observability Gramian $Q \in \mathbb{C}^{n \times n}$ is defined by (Gawronski (2004))

$$Q = \int_0^\infty \exp(A^H t) C^H C \exp(At) \mathrm{d}t \tag{2}$$

and can be calculated by the Lyapunov equation

$$A^H Q + QA + C^H C = 0 \tag{3}$$

It measures the output excitation if the system is in state x_0 at time $t = 0$. The expression $x_0^H Q x_0$ gives the total output "energy" generated by the system for $u(t) = 0 \;\; \forall t > 0$ which is defined by the integral

$$\int_0^\infty y^H(t) \cdot y(t) \mathrm{d}t = x_0^H Q x_0 \tag{4}$$

It seems intuitive to use this quadratic form for the construction of a scalar product or correlation measure.

3.2 Scalar Product for Input Correlation

We now want to measure the dissimilarity of the output signals that can be generated by two inputs to the system. For this purpose, the scalar product

$$\langle v_i, v_j \rangle_G = \int_0^\infty y_i^H(t) y_j(t) \mathrm{d}t \tag{5}$$

will be used where the signal $y_i(t)$ is generated by element v_i from the input space and $y_j(t)$ is generated by element v_j from the input space for $x(0) = 0$. Note that the scalar product is specific to a system G. The following lemma permits to compute the above scalar product for elements of the input space v_i and v_j. An appropriate test-signal must be chosen for the input. The Dirac-signal is appropriate for this purpose since it has a full and uniform frequency spectrum. The following lemma introduces the output controllability matrix $M \in \mathbb{C}^{m \times m}$ which is the output controllability Gramian of the dual system(Halvarsson (2008)). It must not be confused with the output controllability Gramian.

Lemma 1. Consider system (1) with observability Gramian (2). Let $y_i(t)$ be the output signal generated from a dirac input signal $u_i(t) = v_i \delta(t)$ and $y_j(t)$ the output signal generated from another dirac input signal $u_j(t) = v_j \delta(t)$. Then, the scalar product of both outputs satisfies

$$\int_0^\infty y_i^H(t) y_j(t) \mathrm{d}t = v_i^H M v_j \tag{6}$$

with the output controllability matrix

$$M = B^H Q B \tag{7}$$

hermitian and positive semidefinite.

Proof. The solution for the output of system (1) is

$$y_k(t) = C \cdot \int_{-\infty}^t \exp(A(t-\tau)) B u_k(\tau) \mathrm{d}\tau \tag{8}$$

With the inputs $u_k(\tau) = v_k \delta(\tau)$, one obtains

$$y_k(t) = \begin{cases} C \cdot \exp(At) B v_k & \text{if } t > 0 \\ 0 & \text{else} \end{cases} \tag{9}$$

Substituting this (for $= i, j$) into the left side of (6) gives

$$\int_0^\infty y_i^H(t) y_j(t) \; \mathrm{d}t \tag{10}$$

$$= \int_0^\infty v_i^H B^H \exp(A^H t) C^H C \exp(At) B v_j \; \mathrm{d}t \tag{11}$$

$$= v_i^H B^H \underbrace{\int_0^\infty \exp(A^H t) C^H C \exp(At) \mathrm{d}t}_{=Q} \cdot B v_j \tag{12}$$

The lemma follows direcly from the definition of the observability Gramian (2). Symmetry and definiteness of M follow from positive definiteness of Q.

The scalar product induces the norm $\| v \|_G = \sqrt{\langle v, v \rangle_G}$. This norm represents an upper bound for the output energy per unit of input energy that can be generated using a specific input. The following relation to the H_2-Norm of the system should be noted (Toscano (2013)):

$$\| G \|_2 = \sqrt{\mathrm{trace}(M)} = \sqrt{\sum_{i=1}^n \sigma_i(M)} \tag{13}$$

with $\sigma_i(M)$ being the singular values of M.