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A Global Optimal Control Methodology and its Application to a Mobile Robot Model

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Abstract: A global optimal control algorithm is developed and applied to an omni-directional mobile robot model. The aim is to search and find the most intense signal source among other signal sources in the operation region of the robot. In other words, the control problem is to find the global extremum point when there are local extremas. The locations of the signal sources are unknown and it is assumed that the signal magnitudes are maximum at the sources and their magnitudes are decreasing away from the sources. The distribution characteristics of the signals are unknown, i.e. the gradients of the signal distribution functions are unknown. The control algorithm also doesn't need any position measurement of the robot itself. Only the signal magnitude should be measured via a sensor mounted on the robot. The simulation study shows the performance of the controller.

Keywords: Global optimization, mobile robot, extremum seeking.

1. INTRODUCTION

When a mobile robot operates in an unknown terrain with no GPS signal and no inertial measurements, it should use an extremum seeking algorithm to search and find the target point such as a signal source. Various types of extremum seeking algorithms are developed in the literature. In the perturbation based extremum seeking algorithms (Ghaffari et al., 2014; Krstic and Wang, 2000; Zhang et al., 2007) disturb and observe methods are studied by adding a perturbation to the search signal. According to the effect on the system output, the search signal is increased or decreased. In sliding mode based technique (Dincmen et al., 2014; Dincmen and Guvenc, 2012; Drakunov et al., 1995; Fu and Ozguner, 2011; Haskara et al., 2000), a sliding surface is selected such that when the system is in sliding mode, the states move towards the a priori unknown optimum operation point. The numerical optimization based schemes (Vweza et al., 2015; Zhang and Ordonez, 2007) use various iteration algorithms to find the optimum operation point. The iteration method finds the target state and a state regulator manage the system follow this new state. In the fourth group of extremum seeking methods (Guay and Dochain, 2015; Guay, 2014), the search procedure is accomplished via adaptive gradient estimation techniques.

In this paper, a new sliding mode based extremum seeking algorithm is developed where the sliding mode technique is utilized to estimate the gradient of the unknown signal distribution function. In that way, the proposed methodology embodies the characteristics of the second and fourth extremum seeking algorithms presented above. The algorithm is applied to an omni-directional mobile robot signal source seeking problem. Omni-directional mobile robots (Barreto et al., 2014; Kim and Kim, 2014; Li et al., 2015) are holonomic robots which have wheels with free rollers.

When there are multiple signal sources in the operation region of the mobile robot, and if the aim is to find the most intense signal source among the other sources, then this is a global optimization problem. The control algorithm developed in this paper will seek the global maximum point when there are local extremas. It doesn't need to know the signal distribution characteristics, the locations of the signal sources and position of the robot itself. Henceforth, it can be used in the missions where the robot moves in an unknown terrain with no GPS and no inertial measurements. Only the magnitude of the signal should be measured via a sensor mounted on the robot itself.

The rest of the paper is organized as follows: In Section 2, the control algorithm is introduced. Mobile robot model is given in Section 3. Simulation study in Section 4 shows the performance of the control algorithm. The paper ends with conclusions in Section 5

2. CONTROL ALGORITHM

Change of the signal value with respect to the inertial coordinates x,y is denoted here as a nonlinear performance function $J(x,y)$. The controller doesn't know this function i.e. the gradients of the function and its extremum points are unknown. Only its magnitude can be measured via a sensor on the robot. The aim is to find the global extremum point of $J(x,y)$.

2.1 Gradient Estimation Algorithm

For the gradient estimator, a sliding surface variable s is defined as

$$s = J(x,y) + z_1 + z_2 \,, \tag{1}$$

where the time derivatives of the variables z_1 and z_2 are defined as $\dot{z}_1 = -\dot{x}u_1$ and $\dot{z}_2 = -\dot{y}u_2$. Here, u_1 and u_2 are

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discontinuous functions of s and they will be defined later. The time derivative of (1) can be written as

$$\dot{s} = \frac{\partial J}{\partial x}\dot{x} + \frac{\partial J}{\partial y}\dot{y} - \dot{x}u_1 - \dot{y}u_2 . \tag{2}$$

Now, for some time interval Δt_1, when only the motion on x axis is allowed, then, since $\dot{y} = 0$ during this interval, the equation given in (2) becomes

$$\dot{s} = \frac{\partial J}{\partial x}\dot{x} - \dot{x}u_1 . \tag{3}$$

Since the motion is maintained on x axis during Δt_1, then $\dot{x} \neq 0$ will be true in (3). By remembering that u_1 is a discontinuous function of s, then, when $\dot{s} = 0$ is accomplished, from (3), the equivalent value of the discontinuous function u_1 will be equal to

$$u_{1eq} = \left(\frac{\partial J}{\partial x}\right)_{est} . \tag{4}$$

In other words, estimation of the gradient with respect to x will be accomplished by calculating the equivalent value of the discontinuous function u_1. To obtain the equivalent value of u_1, a low-pass filter can be used as

$$\left(\frac{\partial J}{\partial x}\right)_{est} = u_{1eq} = \frac{1}{\tau_1 p + 1}u_1 , \tag{5}$$

where τ_1 is the filter time constant and p is the complex variable of the Laplace transform. So, in (5), $1/(\tau_1 p + 1)$ is the transfer function of a first order system, which is the low-pass filter here. The rationale using a low-pass filter to obtain the equivalent value of the discontinuous function u_1 can be explained as follows: Since u_1 is a discontinuous function of s, during sliding mode, i.e. when $\dot{s} = 0$, it will oscillate with high frequency. The mean value (equivalent value) of these oscillations can be derived by filtering out the high frequency component by using a low-pass filter.

For another time interval Δt_2, when only motion on y axis is allowed, then, since $\dot{x} = 0$ during this interval, the equation given in (2) becomes

$$\dot{s} = \frac{\partial J}{\partial y}\dot{y} - \dot{y}u_2 . \tag{6}$$

Again, since during Δt_2 the motion on y axis is maintained, it is true that $\dot{y} \neq 0$. If $\dot{s} = 0$ is accomplished in (6), then the equivalent value of the discontinuous function u_2 will be equal to

$$u_{2eq} = \left(\frac{\partial J}{\partial y}\right)_{est} , \tag{7}$$

which is the estimate of the gradient with respect to y. To obtain the equivalent value of the discontinuous function u_2, a low pass filter similar to (5) can be used as

$$\left(\frac{\partial J}{\partial y}\right)_{est} = u_{2eq} = \frac{1}{\tau_2 p + 1}u_2 . \tag{8}$$

So, the gradients of the performance function with respect to x and y can be estimated as u_{1eq} and u_{2eq}. It should be reminded that the shape of the performance function $J(x,y)$ is unknown. Henceforth, the exact gradient values $\partial J/\partial x$ and $\partial J/\partial y$ are unknown. The controller will use only the estimated values of the gradients, which are u_{1eq} and u_{2eq} calculated from (5) and (8).

2.2 Gradient Climbing Rule

Desired velocity values on x and y axes can be calculated by using the estimated gradient values according to the gradient climbing rules of

$$\dot{x}_{set} = V_1 \operatorname{sgn}^*\left(\frac{\partial J}{\partial x}\right)_{est} = V_1 \operatorname{sgn}^*(u_{1eq}) , \tag{9}$$

$$\dot{y}_{set} = V_2 \operatorname{sgn}^*\left(\frac{\partial J}{\partial y}\right)_{est} = V_2 \operatorname{sgn}^*(u_{2eq}) , \tag{10}$$

where V_1 and V_2 are positive constants and representing the step sizes of the motion during gradient climbing. The function sgn* is defined as

$$sgn^*(\varepsilon) = \begin{cases} 1 & \text{if } \varepsilon \geq 0 \\ -1 & \text{otherwise} \end{cases} . \tag{11}$$

According to (9), (10) and (11), the reference velocities take values of $\dot{x}_{set} = \pm V_1$ and $\dot{y}_{set} = \pm V_2$. The equivalent values of the discontinuous functions u_1 and u_2 are the estimated gradients as in (4) and (7). The gradient estimation will be valid if $\dot{s} = 0$ in (3) and (6). The necessary conditions for accomplishing $\dot{s} = 0$ are given in the following subsection.

2.3 Necessary Conditions to Make $\dot{s} = 0$

For the interval of Δt_1, where the motion is only on x axis, the sliding surface dynamics was given in (3). Here, if the discontinuous function u_1 is selected as

$$u_1 = M_1 \operatorname{sgn}\left[\sin\left(\frac{\pi s}{\gamma_1}\right)\right] , \tag{12}$$

where M_1 and γ_1 are positive constants, "sgn" is the signum function and "sin" is the sinusoidal function, then, if M_1 is chosen to satisfy the condition

$$M_1 > \left|\frac{\partial J}{\partial x}\right|_{\max} , \tag{13}$$

then after a finite time interval, $\dot{s} = 0$ will be accomplished.

Proof: By integrating (3) with (12), one can obtain

$$\dot{s} = \frac{\partial J}{\partial x}\dot{x} - \dot{x}M_1 \operatorname{sgn}\left[\sin\left(\frac{\pi s}{\gamma_1}\right)\right] . \tag{14}$$

By assuming perfect velocity set point tracking, from (9) one can write $\dot{x} = \dot{x}_{set} = \pm V_1$. When $\dot{x} = V_1$, (14) can be written as

$$\dot{s} = \frac{\partial J}{\partial x}V_1 - V_1 M_1 \operatorname{sgn}\left[\sin\left(\frac{\pi s}{\gamma_1}\right)\right] . \tag{15}$$