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Iterative Control for Periodic Tasks with Robustness Considerations, Applied to a Nanopositioning Stage ⋆

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Abstract: Nanopositioning stages are an example of motion systems that are required to accurately perform high frequency repetitive scanning motions. The tracking performance can be significantly increased by iteratively updating a feedforward input by using a nonparametric inverse plant model. However, in this paper it is shown that current approaches lack systematic robustness considerations and are suffering from limited design freedom to enforce satisfying convergence behavior. Therefore, inspired by the existing Iterative Learning Control approach, robustness is added to the existing methods to enable the desired convergence behavior. This results in the Robust Iterative Inversion-based Control method, whose potential for superior convergence is experimentally verified on a Nanopositioning system.

Keywords: Periodic motion, Iterative methods, Robustness, Nanopositioning, Nonparametric

1. INTRODUCTION

The demand for increased accuracy and speed in precision applications has led to the adoption of dedicated feedforward control inputs (Devasia et al., 2007; Pipeleers et al., 2009). For example, in Atomic Force Microscopy, a sample is moved relative to a probe with nanometer resolution and scan frequencies reaching hundreds of Hertz. Since the resonance frequencies of the stages are not significantly higher than desired scan rate, achieving an acceptable bandwidth is a non-trivial task, as argued in Fleming and Leang (2014). Fortunately, since the setpoint is a periodically repeating trajectory, the desired scanning speeds can be achieved by iteratively updating a feedforward input signal as is shown in Tien et al. (2005). For such an iterative control method to be successful, it should generally posses the following three features;

(1) the converged performance should meet the desired level of accuracy;
(2) the convergence speed of the iterative solution should be within an acceptable number of iterations;
(3) and the algorithm should be robust against perturbations to the controlled plant.

Several iterative methods have been proposed that aim to increase the tracking performance of motion systems that perform periodic tasks, while satisfying these criteria. In Tien et al. (2005) the Iterative Inversion-based Control (IIC) method was proposed that employs an inverse plant model to iteratively update the feedforward input. This approach is reported to lead to a significant increase in tracking performance in case the plant model is sufficiently accurate. However, it was argued that the convergence rate can be prohibitively slow in case the model mismatch is large. Consequently, an extension to this method (EIIC) is made in Zou et al. (2007), which relaxes the convergence criteria to some extent but the restrictions imposed by the required accuracy of the inverse plant model remain. The Model-less IIC (MIIC) as approach, as presented in Kim (2008) aims to remove the convergence criteria completely by estimating the inverse plant model in the iterative process. Promising results are reported in Bechhoefer (2008) for the case in which the effects of nonlinearities such as hysteresis are limited.

Although these important developments in IIC have led to significantly increased tracking performance, robustness aspects are not considered systematically. Moreover, the connection to Repetitive Control (RC) and Iterative Learning Control (ILC) is not yet established and with that, similar design guidelines have been left largely unformulated. Therefore, the aim of this paper is to fill this gap and experimentally verify the proposed approach.

In this paper, RC, ILC and IIC are formulated in a general lifted signal description. This unified formulation shows that the IIC is very similar to ILC and consequently the ILC design guidelines can be modified to systematically treat robustness and convergence rate aspects

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in IIC methods. Thus, this paper contains the following contributions.

C1. In section 2 a systematic comparison of the RC, ILC and IIC methods in a unifying description is provided.
C2. In section 3 the IIC method is extended, based on existing ILC results and design guidelines are provided.
C3. In section 4 the proposed method is experimentally validated on a nanopositioning system.

2. A UNIFYING ANALYSIS

In this section, the lifted signal description as first presented in Bamieh et al. (1991) and the problem of periodic disturbance rejection are introduced, which enable a unifying time-domain analysis of the RC, ILC and IIC methods.

2.1 Rejection of periodic disturbances in a lifted framework

A linear time invariant (LTI), single-input-single-output (SISO), stable discrete time system, $G(z)$ is considered, where $z \in \mathbb{C}$ is a complex indeterminate. The evolution of the output $y(k) \in \mathbb{R}$, subjected to the input $u(k) \in \mathbb{R}$ and initial state $x(t_0) = x_0$, is described by the state space equations,

$$x(k+1) = Ax(k) + Bu(k), \quad (1a)$$
$$y(k) = Cx(k) + Du(k), \quad (1b)$$

where (A, B, C, D) is a realization of G and $x(k) \in \mathbb{R}^n$ is the state vector. Now define N as the number of samples of a single trial and denote the trial number by i. A lifted signal $\bar{s}$ is now defined as the discrete time sequence of the signal $s(k)$ during a single trial, whose elements are stored in a column, i.e.,

$$\bar{s}_i \triangleq \begin{bmatrix} s(Ni) \\ s(Ni+1) \\ \vdots \\ s(Ni+N-1) \end{bmatrix}. \quad (2)$$

By evaluating the state space equations as given by (1a) and (1b), it can be found that the lifted system G_l can be represented as,

$$G_l \triangleq \begin{cases} x(Ni+N) = Fx(Ni) + G\bar{u}_i, & (3a) \\ \bar{y}_i = Hx(Ni) + J\bar{u}_i, & (3b) \end{cases}$$

$$\left[\begin{array}{c|c} F & G \\ \hline H & J \end{array}\right] = \left[\begin{array}{c|ccccc} A^N & A^{N-1}B & \dots & AB & B \\ \hline C & h(0) & 0 & \dots & 0 \\ CA & h(1) & h(0) & & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ CA^{N-1} & h(N-1) & \dots & h(1) & h(0) \end{array}\right], \quad (4)$$

where $h(k)$ are the Markov parameters,

$$h(k) = \begin{cases} D & k = 0 \\ CA^{k-1}B & k \geq 1 \end{cases}. \quad (5)$$

This system representation is key in the analysis of periodic signals that are periodic with N samples. Note that for these signals holds that $\bar{s}_i = \bar{s} \; \forall j$, since $s(k) = s(k+N) \; \forall k$. The problem of Iterative Periodic Disturbance Rejection (IPDR) is now formulated as follows.

Problem 1. (IPDR, time domain). Consider a periodic disturbance $\bar{r}$ and define the lifted tracking error as,

$$\bar{e}_i \triangleq \bar{r} - \bar{y}_i. \quad (6)$$

Then, (iteratively) find $\bar{u}_i$ such that $\lim_{i\to\infty} \bar{e}_i = 0$.

2.2 Repetitive Control

The RC method aims to solve the IPDR problem in case the state x simply results from the past inputs during the previous trial as is reflected by equation (3a). This is typically the case for systems that perform continuous periodic tasks such as hard disk drives and power electronics. Hence, the RC method aims to provide $\bar{u}_i$ in case $\bar{e}_i$ is given by (3a), (3b) and (6). See for example Hara et al. (1988) and Roover et al. (2000).

2.3 Iterative Learning Control

The ILC approach considers the case for which the state x resets after each trial, i.e. $x(Ni) = \hat{x}$, which can be assumed to be zero without loss of generality, see for example Bristow et al. (2006). This is typically the case for batch-to-batch processes such as pick-and-place and printing tasks. In this case, equations (3a) and (3b), combined with (6) reduce to,

$$\bar{e}_i = \bar{r} - J\bar{u}_i, \quad (7)$$

where J is also known as the discrete impulse response matrix. The ILC method aims to provide $\bar{u}_i$ by iteratively updating the input as,

$$\bar{u}_{i+1} = Q\bar{u}_i + L\bar{e}_i. \quad (8)$$

Here, L is the learning filter which is often taken to be such that it approximates the inverse of J, while considering causality and minimum-phase aspects. Robustness is introduced by sensibly shaping Q in which case it is no longer equal to the unity matrix, see for example Tousain et al. (2001), van de Wijdeven et al. (2009).

2.4 Iterative Inversion-based Control and related methods

The IIC and related approaches aim to solve the IPDR problem by means of a frequency domain approach. In these methods, it is assumed that the input signal u is periodic with a periodicity of N samples, i.e., $u(k) = u(k+N) \; \forall k$, and this signal has been driving the system for an infinite time from an arbitrary initial condition. Consequently, the periodic output y is in steady state and is given in the frequency domain as,

$$Y(\omega) = G(e^{j\omega})U(\omega), \quad \omega \in \Omega, \quad (9)$$

$$\Omega = \left\{ \omega \in \mathbb{R} \;\middle|\; \omega = \tfrac{2\pi}{N}k, \; k = 0, \dots, N-1 \right\}. \quad (10)$$

Here, Y and U are the Fourier coefficients of the output y and the input sequence u, respectively and Ω is the discrete frequency grid. The IPDR problem can now be formulated in the frequency domain as follows.

Problem 2. (IPDR, frequency domain). Consider the Fourier coefficients $R(\omega)$ of the periodic disturbance reference signal $\bar{r}$ and define the tracking error as,

$$E(\omega) \triangleq R(\omega) - Y(\omega), \quad \omega \in \Omega, \quad (11)$$

Then, (iteratively) find $U_i(\omega)$ such that $\lim_{i\to\infty} E_i(\omega) = 0 \; \forall \omega \in \Omega$.

Note that the frequency domain formulation is more specific since it assumes that the system is in steady state.