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Stability Criterion for Voltage Stability Study of Distributed Generators ⋆

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Abstract: This paper proposes a stability criterion for the study of distributed generators equipped with local reactive power regulations. Existing formal methods propose a memory-consuming analysis with some practical issues while dealing with large-scale systems. To cope with this, a novel analytic stability criterion is established in this work. Firstly, a necessary condition for system stability is demonstrated for a feeder hosting a single generator. The approach is illustrated on a real medium voltage feeder. Then, a conjecture is proposed to study the stability of feeders hosting several generators equipped with reactive power regulations. Conjecture validity is proved over several theoretical grids hosting up to four distributed generators thanks to simulations.

Keywords: Voltage stability, Hybrid systems, Piecewise linear systems, Electric power systems, Reactive power.

1. INTRODUCTION

During the past decades, the share of generators connected to the distribution grid – the distributed generators (DGs) – has severely increased thereby strongly modifying distribution grids behavior (Azmy and Erlich, 2005). As one of the consequences of this change, the voltage along distribution feeders hosting generation has increased (Dai and Baghzouz, 2003). To cope with this, distribution grid operators (DSOs) have imagined many solutions to maintain the voltage within acceptable limits. The classic solution consists in reinforcing the network thus mitigating voltage issues but with a subsequent cost. In France, the main DSO (ERDF) estimates the cost of photovoltaic power connection to be up to 300 k€/MW in 2030. In order to try to avoid – or at least to postpone – such investments, numerous alternatives to network reinforcement have been investigated in the literature (Dutrieux et al., 2015). Among these, local control laws of DGs reactive power (Q) with respect to their voltage (U) have particularly drawn attention such as in Unger et al. (2013) or Turitsyn et al. (2011). Indeed, such a control law offers to mitigate voltage issues without limiting the DG's injection of active power. Indeed, the DG measures the voltage at its connection point and adapts, in real time, its reactive power set point according to a given lookup table. The shape of $Q(U)$ regulation adopted by ERDF the French DSO (Witkowski et al., 2013) is shown in Fig. 1.

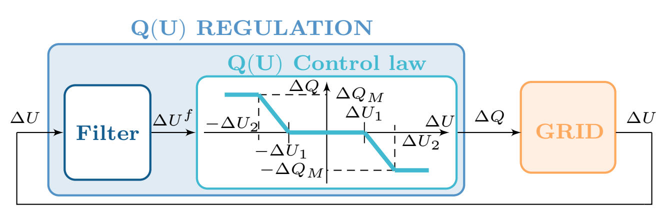

Fig. 1. General structure of the system under study

As it can be seen in Fig. 1, this is a closed-loop regulation and so may endanger voltage stability. Even if there exists a lot of work on voltage control by reactive power management with inverters, very few have studied the stability of a grid hosting many DGs equipped with $Q(U)$ regulations. This can be explained as such studies raise considerable challenges due to the non-linearity of the $Q(U)$ control law. For example, the control law considered in this work is piecewise affine with five operating modes (Fig. 1). In order to assess the stability of a grid hosting one or several

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DGs equipped with $Q(U)$ regulations, simulations and experimentations have been carried out.

In empirical studies presented by Beauné et al. (2014) and Esslinger and Witzmann (2013), $Q(U)$ regulations do not cause any system instability. However, other works have drawn contradictory conclusions. Stetz (2014) presents a simulation work evidencing unstable operating points for a grid hosting one DG equipped with a $Q(U)$ regulation. Without a formal stability study of these regulations, no general conclusion can be drawn. Recently, Andren et al. (2015) has proposed an analytic study of the stability of a grid hosting several $Q(U)$ regulations functioning in one linear operating mode. This linearity assumption allows the authors to assess stability by computing the eigenvalues of the system with some limitations on the validity domain. Preliminary work (Cosson et al., 2015a) develops a new stability study method coping with the non-linearity of the $Q(U)$ regulation. This method is based on the computation of a discrete abstraction of the system and its refinement thanks to bisimulation calculations. This method performs a formal stability analysis of a piecewise affine hybrid system but at a high computational cost.

The present paper formulates a stability criterion for a distribution grid hosting several $Q(U)$ regulations and so allows us to conclude on system stability while keeping a reduced computational load. This expression lies in the study of the possible commutations between several linear operating modes. An intensive investigation leads to an analytic criterion for the stability of one DG expressing stability limits with respect to regulation and grid parameters. Then, an extrapolation of this result to the grids hosting several DGs is presented.

Section 2 details the studied system and the proposed model. Then, the analytic expression of the stability criterion for one DG is elaborated in Section 3. The next section presents the extrapolation of the stability criterion to grids hosting several DGs and simulation results validating the criteron for realistic case-studies. Lastly, the conclusions of this work are developed in Section 5.

2. SYSTEM MODELING

A medium-voltage feeder, connecting several consumers and n generators is considered. All n DGs are supposed to be equipped with the same $Q(U)$ regulation. As presented in Fig. 1, these regulations measure and filter the voltage magnitude at the DGs buses $\mathbf{U}(k) \in \mathbb{R}^n$. The measurement filter is considered to be a discrete-time first-order low-pass filter with a sample time $T_s = 1s$ and a unit gain. Then, the filtered voltage magnitude $\mathbf{U^f}(k) \in \mathbb{R}^n$ is converted into a reactive power set point $\mathbf{Q}(k) \in \mathbb{R}^n$ through a piecewise affine $Q(U)$ law with five operating modes. The purpose of this work is to study the possible voltage oscillations caused by the $Q(U)$ regulations. If such oscillations exist, they would have a period larger than the sampling time T_s of the regulation, typically approximately of a few seconds. Thus, in order to study these phenomena, the appropriate model is an electromechanical one (Kundur et al., 1994). All electromagnetic phenomena will be modeled in steady-state.

As network lines transients are electromagnetic phenomena (Kundur et al., 1994), they are modeled in steady-state. Thus, the grid behavior can be represented by the power flow equations (Bolognani and Zampieri, 2016). The model of the grid should express the voltage magnitude variation explicitly at the DGs buses $\mathbf{\Delta U}(k) \in \mathcal{U}^n \subset \mathbb{R}^n$ with respect to reactive power changes $\mathbf{\Delta Q}(k) \in \mathcal{Q}^n \subset \mathbb{R}^n$. To do so, a linear approximation of the power flow equations is computed around the operation point defined by $\mathbf{Q} = \mathbf{0}$

$$\mathbf{\Delta U}(k) = \mathbf{K_Q} \mathbf{\Delta Q}(k) + \mathbf{K_d} \mathbf{\Delta U_d}(k) \qquad (1)$$

where $\mathbf{\Delta U_d}(k) = [\Delta U_{d_1}(k), \cdots, \Delta U_{d_m}(k)]^T$ is a vector of m disturbance variables. In this model, all non-controlled variables are considered as disturbances such as variations in active power, voltage at the primary substation, consumed power, etc. The matrices $\mathbf{K_Q} \in \mathbb{R}^{n\times n}$ and $\mathbf{K_d} \in \mathbb{R}^{n\times m}$ can be calculated as the sensitivity matrices of the bus voltage magnitude to the variations of the injected reactive power and disturbances.

The DGs are supposed to be connected to the grid through power electronics as it is generally the case (Machowski et al., 2011). The control of reactive power with power electronics converters, such as inverters, can be modeled as a first order low-pass filter with a unit gain and a time constant of a few milliseconds (Machowski et al., 2011). Thus, in this work, the DGs power electronics are modeled in steady-state. Lastly, the DGs behavior is modeled by their $Q(U)$ regulations composed of a measurement filter and a $Q(U)$ control law. The discrete-time first-order low-pass filter is modeled by its state equation with a unit gain and $a \in [0,\ 1[$ equivalent to its time constant and so to filter rapidity.

$$\mathbf{\Delta U^f}(k+1) = a\ \mathbf{\Delta U^f}(k) + (1-a)\ \mathbf{\Delta U}(k) \qquad (2)$$

where $\mathbf{\Delta U^f}(k) \in \mathcal{U}^f \subset \mathbb{R}^n$ is the vector of filtered voltage magnitudes at the DGs buses at time $t = kT_s$. Afterwards, the variation of the reactive power set point $\mathbf{\Delta Q}(k)$ is computed through a piecewise affine function.

$$\mathbf{\Delta Q}(k) = \mathbf{G}(\mathbf{I}(k))\ \mathbf{\Delta U^f}(k)\ +\ \mathbf{F}(\mathbf{I}(k)) \qquad (3)$$

where $\mathbf{I} \in \mathcal{I}^n = \{1,\ldots,5\}^n$ is the vector of the operating modes of each DG at time kT_s such as:

$$\mathbf{\Delta U}^-(\mathbf{I}(k)) \le \mathbf{\Delta U^f}(k) \le \mathbf{\Delta U}^+(\mathbf{I}(k)) \Leftrightarrow \mathbf{\Delta U^f}(k) \in \mathcal{D}_{\mathbf{I}(k)} \qquad (4)$$

where $\mathcal{D}_i$ denotes, for every $i \in \mathcal{I}$, the polyhedron defined by the set of points $u \in \mathbf{U}^f$ such as $\Delta U^-(i) \le u \le \Delta U^+(i)$.

The matrix $\mathbf{G}(\mathbf{I}(k)) \in \mathbb{R}^{n\times n}$ is a diagonal matrix with $g_{jj}(i_j(k))$ being the slope of the $Q(U)$ law of the j-th DG which is in the $i_j(k)$-th operating mode. Each component $f_j(i_j(k))$ of the vector $\mathbf{F}(\mathbf{I}(k)) \in \mathbb{R}^n$ corresponds to the intercept of the j-th DG $Q(U)$ law in mode $i_j(k)$ as indicated in Table 1.

i_j	1	2	3	4	5
g_{jj}	0	$\sigma = \frac{-\Delta Q_M}{\Delta U_2 - \Delta U_1}$	0	σ	0
f_j	ΔQ_M	$\sigma \Delta U_1$	0	$-\sigma \Delta U_1$	$-\Delta Q_M$
ΔU_j^-	$-\Delta U_M$	$-\Delta U_2$	$-\Delta U_1$	ΔU_1	ΔU_2
ΔU_j^+	$-\Delta U_2$	$-\Delta U_1$	ΔU_1	ΔU_2	$+\Delta U_M$

Table 1. Parameters of the $Q(U)$ law of the j-th DG in each operating mode i_j