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Reachability-based Control Synthesis for Power System Stability

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Abstract: We present a nonlinear reachability framework for control of power system transients. The power system consists of synchronous generators, coupled by transmission lines and controlled HVDC links. The approach consists of characterisation of the region of attraction as a backward reachable set of a stable equilibrium point of the power system. Furthermore, the optimal HVDC control as a feedback map is synthesised. As such, the set of the generators' disturbed states that return to a stable equilibrium point under an optimal HVDC control input are computed. The numerical examples include a two area power system with a HVDC link.

Keywords: Power system transient stability, Reachability analysis, Nonlinear optimal control

1. INTRODUCTION

Transient stability refers to the ability of a power system to return to a stable operating condition after being exposed to severe disturbances, such as faults or component losses Sauer and Pai (1998), Kundur (1994). Ensuring transient stability becomes increasingly challenging as transmission systems are operated with an increasing share of intermittent renewable energy sources, causing more power flow fluctuations, reduced system inertia and larger prediction uncertainty of the net power supply and demand.

Given the importance of transient stability, a plethora of approaches have been developed to analyse this problem Pavella et al. (2012). In this brief paper, we state past work most relevant to the work presented here. These include model based approaches that characterise region of attraction of a stable equilibrium point. The region of attraction denotes the set of all states that return to a stable equilibrium after a disturbance. For an autonomous power system, the region of attraction has been estimated using energy based Lyapunov functions (please see Varaiya et al. (1985); Pai (2012) and references therein). Lyapunov's direct method provides a sufficient condition for stability. Thus, in general, it results in conservative estimation of the region of attraction. Consequently, the search for Lyapunov functions which enlarge the region of attraction is ongoing. An alternative approach is the reachability based analysis. This approach was studied for an autonomous nonlinear model of a power system and for a power system with discrete control inputs in Jin et al. (2010); Susuki et al. (2012). In Althoff et al. (2012), the nonlinear swing equations were approximated by linear differential inclusions and linear system reachability was developed to address transient stability.

In this paper, we apply reachability based analysis to stabilise controlled power systems during transients. To improve the stability properties of an autonomous power system, controllable components can be used to influence the dynamic power flows. These components include controllable loads, power system stabilisers, FACTS devices, and controllable generation such as photovoltaic sources or flywheels. We incorporate Voltage Source Converter based High Voltage Direct Current (VSC-HVDC) links. Recent work demonstrated the potential of HVDC control for transient stability, using a linearised model and a model predictive control approach Fuchs et al. (2014). Whereas for small disturbances, linearised power system dynamics can be analysed, for transient stability, disturbances and faults can push the system outside its assumed linearised operating region, and thus we focus on nonlinear models.

Our contributions are threefold. First, we find an exact characterisation of maximal region of attraction for the HVDC controlled power system model. The approach is based on backward reachable set computation of a carefully chosen target set. The target set is a basin of attraction of a stable equilibrium of the autonomous power system and this basin of attraction is derived using the energy function method Münz and Romeres (2013). Second, we apply the reachability framework to synthesise optimal HVDC control policy. These optimal HVDC injections ensure that states reach the stable equilibrium in minimum time starting from any initial condition in the maximal region of attraction. Given that nonlinear reachability method relies on no approximations inducing conservatism to the reachable set characterisation, our result serves as a benchmark for comparison with other methods. Our third contribution is to compare our results with the energy function method in Münz and Romeres (2013) for an autonomous power system. The case studies include a one area and a two area power system with and without controlled HVDC links, thereby capturing a large range of power system phenomena.

The paper is organised as follows. Section 2 presents the system model. Section 3 presents the approach for characterising the region of attraction and control design. Section 4 provides the simulation results of the selected case studies. Section 5 concludes the paper.

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2. POWER SYSTEM MODEL

We first present the general transient power system model and next specialise it to the one and two area power systems. The one and two area power systems serve as qualitative and to some precision also quantitative approximation of a broad range of power system stability phenomena Kundur (1994). In particular, AC line losses or growing oscillations of dominant modes in a meshed power system can be qualitatively modelled with two machines representing the two aggregated areas (two are power system), connected by an AC line representing the meshed AC grid in between. Generator or load losses and especially short-circuit faults, can destabilise a local region deviating from the main system frequency. This phenomenon can be qualitatively modelled with a one area power system consisting of a single machine, representing the aggregated area of machines, connected to an infinite bus, representing the large remaining power system.

2.1 General power system dynamics

In a multi-machine system with n_{G} generators, a model of the power system dynamics is given by the coupled nonlinear swing equations Kundur (1994). For the i'th generator, the associated swing equation is given by

$$\frac{2H_i}{\omega_0}\frac{\mathrm{d}^2\delta_i}{\mathrm{d}t^2} = P_{\mathrm{m},i} - P_{\mathrm{e,i}} - \frac{D_i}{\omega_0}\cdot\frac{\mathrm{d}\delta_i}{\mathrm{d}t} \tag{1}$$

with the generator angle δ_i and its time derivative expressed in a reference frame rotating at the nominal frequency ω_0. The system parameters $\{P_{\mathrm{m},i}, H_i, D_i\}$, denote the mechanical generator power, the inertia constant and the damping constant of the generator.

The electrical power $P_{\mathrm{e,i}}$, injected by the i'th generator into the grid, is a nonlinear function of the generator angles

$$P_{\mathrm{e,i}} = \mathrm{Re}\ (V_{\mathrm{G},i} Y_i^T V_{\mathrm{G}}), \tag{2}$$

with the vector of complex bus voltages V_{G} defined as

$$V_{\mathrm{G}} = [V_{\mathrm{G},1}, V_{\mathrm{G},1}, ..., V_{\mathrm{G},n_{\mathrm{G}}}]^T, \qquad V_{\mathrm{G},i} = 1\cdot e^{j\delta_i}.$$

The parameter Y_i^T is the i'th row of the power systems' admittance matrix $Y \in \mathbb{C}^{n_{\mathrm{G}}\times n_{\mathrm{G}}}$, and depends on the underlying transmission grid, the impedance loads connected to each bus and additional shunt elements. In this work we assume constant voltages normalised at magnitude one.

The electrical power in (1) models the case of an autonomous power system. In case of a controlled power system, additional terms contribute to the electrical power injection into the network as will be described.

2.2 Autonomous one and two area power system

For the case of $n_{\mathrm{G}} = 2$ machines, (1)-(2) can be transformed and written in state space form as

$$\dot{x} = \underbrace{\begin{bmatrix} \omega_0\cdot(x_2 - x_3) \\ \frac{1}{2H_1}\cdot\Big(P_1 - P_{12}\cdot\sin(x_1+\alpha) - D_1 x_2\Big) \\ \frac{1}{2H_2}\cdot\Big(P_2 + P_{12}\cdot\sin(x_1-\alpha) - D_2 x_3\Big) \end{bmatrix}}_{=:f_2^{\mathrm{aut}}(x)} \tag{3}$$

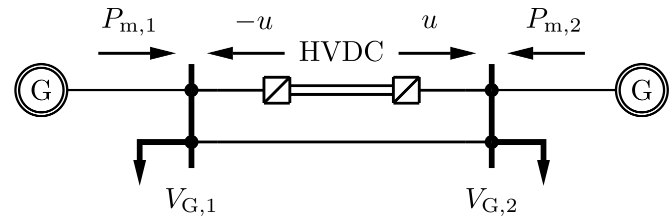

Fig. 1. Two area power system connected by an AC link and an HVDC link.

with the three dynamic states $x = [x_1, x_2, x_3]^T$ defined as

$$x_1 = \delta_1 - \delta_2, \qquad x_2 = \frac{1}{\omega_0}\frac{\mathrm{d}\delta_1}{\mathrm{d}t}, \qquad x_3 = \frac{1}{\omega_0}\frac{\mathrm{d}\delta_2}{\mathrm{d}t}.$$

The fourth dynamic state after transformation of (1) is $x_4 = \delta_1 + \delta_2$. This state assures retractability of the individual angles δ_1 and δ_2, but is not required for transient stability analysis. Thus, it is not included to reduce the computational effort in reachability based analysis and control synthesis. The parameters $\{P_1, P_2, P_{12}, \alpha\}$ are computed from the original parameters $\{P_{\mathrm{m},1}, P_{\mathrm{m},2}, Y\}$.

The one area power system is a special case of the two area power system, with an infinite bus instead of a generator connected to the second node. This is a voltage source of constant voltage angle and frequency and is equivalent to

$$\delta_2 = \dot{\delta}_2 = 0,$$

With $x_1 = \delta_1$, $x_2 = \frac{1}{\omega_0}\frac{\mathrm{d}\delta_1}{\mathrm{d}t}$, (3) simplifies to

$$\dot{x} = \underbrace{\begin{bmatrix} \omega_0\cdot x_2 \\ \frac{1}{2H_1}\cdot\Big(P_1 - P_{12}\cdot\sin(x_1+\alpha) - D_1 x_2\Big) \end{bmatrix}}_{=:f_1^{\mathrm{aut}}(x)} \tag{4}$$

2.3 Controlled one and two area power system

A main purpose of this paper is maximising region of attraction for power systems with actively controlled components. For the applications in this paper, an HVDC link is connected to both generator buses as illustrated in Fig. 1. In addition to its nominal power transmission, the HVDC link allows active control of the power injections at both terminals. As a result, the new system dynamics are

$$\dot{x} = f_2^{\mathrm{aut}}(x) + \left[\ 0,\ \frac{1}{2H_1},\ \frac{-1}{2H_2}\ \right]^T u, \tag{5}$$

with the control input u modelling the HVDC power transmitted from area 1 to area 2, and $f_2^{\mathrm{aut}}(x)$ defined in (3). The internal HVDC dynamics and the power losses of the HVDC link are neglected in this model. The control input has to lie within the bounds

$$u \in U := [-P_{\mathrm{HVDC}}, P_{\mathrm{HVDC}}],$$

where P_{HVDC} is the HVDC power rating. Note that the formulation with the additive input implies that the input could also be provided by any other device that allows direct control of the power injections, such as controllable loads, photovoltaic sources or FACTS devices.

In the special case of the one area power system, the model with HVDC control input is obtained as

$$\dot{x} = f_1^{\mathrm{aut}}(x) + \left[\ 0,\ \frac{1}{2H_1}\ \right]^T u, \tag{6}$$

and with the same input set U.