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Controlled Chaos Based on Tellegen's Principle in Electric Power Systems - Basic Approach

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Abstract: The paper deals with a new problem of physical correctness detection in the area of strictly causal system representations. The proposed approach to the problem solution is based on generalization of Tellegen's theorem well known from electrical engineering. The novelty of this approach is that it is based on the abstract state space energy for real linear, nonlinear and chaotic systems e.g. electrical circuits and power lines with distributed parameters. Consequently, mathematically as well as physically correct results are obtained. Some known and often used system representation structures are discussed from the developed point of view as an addition. The examples are also included.

Keywords: Chaos in Systems, Nonlinear Systems, Simulations, State space energy, Tellegen

1. INTRODUCTION

It is familiar that there are two basic approaches to system modeling. The first one consists in using mathematical formulas and physical tools (a causality principle, different forms of conservation laws, power balance relations, etc.) in order to describe appropriate system behavior. It has successfully been used in many fields of science and engineering so far. However, there are also situations where physical laws are not known or cannot be expressed in a proper mathematically exact form. In that case the second basic approach to system modeling can be turned. It is based on identification methods working in terms of experimentally gained data [Willems, J.C], [Khalil, H.K]. It is possible to divide the methods into two groups: parametric and non-parametric, respectively. If any prior information about a system structure is not assumed then non-parametric methods are used for system identification. On the other hand, imagine that a physical structure of an investigated system is known then parametric methods will be used and subsequently more adequate results should be expected [Mayer, D. and Hrusak, J]. Unfortunately, any reliable explicit knowledge about a physical system structure is more likely an exception than a rule. Therefore, a system structure is mostly chosen on behalf of heuristic arguments and then it is verified whether obtained quantitative results are not in conflict with obvious qualitative expectations concerning regular system behavior and/or results of additional experiments performed on a real system. The main aim of the contribution is to formulate a fundamental problem of physical correctness detection of system representations and in the sequel propose its possible solution. The method starts from the assumption that any physically correct system representation should not be at variance with not only measured data but also a form of an energy conservation principle. It is shown in the paper that introducing the principle as the attribute of a causal system representation seems to be the most natural way as it can be done [Kalman, R.E], [MacFarlane, AG.J], also the energy approach for linear, nonlinear and chaotic system is shown.

2. TELLEGEN'S THOREM AND ITS GENERLIZATION

In order to explain essential features of the theorem [Bostan, A. et al], consider an arbitrarily connected electrical network with n components and choose associated reference directions for branch voltages v_k and currents i_k. Let Kirchhoff's laws be given by the following equations:

$$Ai(t) = 0; \qquad Bv(t) = 0 \tag{1}$$

where A is a node incidence matrix, B is a loop incidence matrix and *i(t), v(t)* are defined as follows:

$$i(t) = \left[i_1(t), i_2, ..., i_n(t)\right]^T; \; v(t) = \left[v_1(t), v_2(t), ... \, v_n(t)\right]^T \tag{2}$$

Let the vectors *i(t), v(t)* be the elements of an Euclidean space E_n and invoke the inner product:

$$\langle i(t), v(t)\rangle = \sum_{k=1}^{b} i_k(t) v_k(t) \tag{3}$$

Let I be the set of all the vectors $i(t)$ and V the set of all the vectors $v(t)$ satisfying the equations (1).

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Theorem 1: (Tellegen's theorem) If $i(t)\in I$ and $v(t)\in V$ then it holds that:

$$\langle i(t), v(t)\rangle = 0 \tag{4}$$

It is worth noticing a close relation between physical correctness and Tellegen's theorem. It is also important to realize that the branch currents and voltages are chosen arbitrarily complied only with Kirchhoff's laws. It implies that different sets $\overline{I}$, $\overline{V}$ of the branch currents and voltages satisfying the laws can be selected and the relation:

$$\langle \overline{i}(t), \overline{v}(t)\rangle = 0,\ \overline{i}(t)\in\overline{I},\ \overline{v}(t)\in\overline{V} \tag{5}$$

still holds. The last deduction will be used later as motivation for introducing a group of system equivalence transformations on which generalization of Tellegen's theorem is based [Mayer, D, 1970], [Ramachandran, R.P].

2.1 Generalized Tellegen's principle

Consider the representation $R(S)$ of a system S in the form

$$R(S): \frac{dz(t)}{dt} = f\left[z(t), u(t)\right] \tag{6}$$

where $z(t)\in Z$ is a state, $Z\subset R^n$ is a smooth manifold and f: $Z\rightarrow R^n$ is a smooth vector field parameterized by an input $u(t)$. Let $E : Z \rightarrow R$ be a smooth scalar field. It is well known that the Lie derivative of the scalar field E with respect to the vector field f is defined as follows [Hrusak, J. et al. 2004]:

$$\begin{aligned} L_f\left\{E\left[z(t)\right]\right\} &= \left\langle dE\left[z(t)\right], f\left[z(t),u(t)\right]\right\rangle \\ &= \sum_{i=1}^{n} \frac{\partial E\left[z(t)\right]}{\partial z_i(t)} f_i\left[z(t),u(t)\right] \end{aligned} \tag{7}$$

Theorem 2: (Generalized Tellegen's principle)

$$\begin{aligned} &\exists E, f, E\left[z(t)\right] = \sum_{i=1}^{n} E_i\left[z_i(t)\right], \\ &\frac{dz(t)}{dt} = f\left[z(t),u(t)\right] : L_f\left\{E\left[z(t)\right]\right\} = 0 \end{aligned} \tag{8}$$

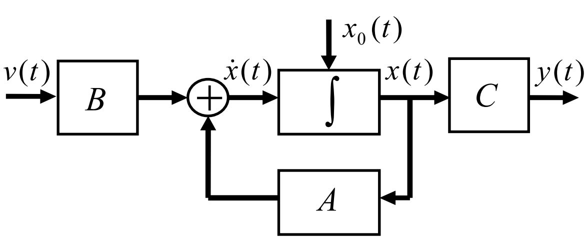

Fig. 1 Block diagram of open loop system (without state or output feedback) with input *v(t)*, output *y(t),* state *x(t)*, initial conditions $x_0(t)$ and matrices *A*, *B*, *C*

System representation structure (system transformation from $z(t)\rightarrow x(t)$, state equivalent representations)

$$\exists \varphi, T, T^{-1}, x(t) = T\left[z(t)\right]; u(t) = \varphi\left[v(t), x(t)\right]; \tag{9}$$

Consider a class of state equivalent representations (see block diagram of Fig.1) described by equations (9) – (11) and structure shown in Fig. 2 [Hrusak, J. et al. 2006]

$$\frac{dx(t)}{dt} = A\cdot x(t) + B\cdot v(t);\ y(t) = C\cdot x(t) \tag{10}$$

$$A = \begin{bmatrix} -\alpha_{11} & \alpha_2 & 0 & 0 & \cdots & 0 & 0 \\ -\alpha_2 & \alpha_{22} & \alpha_3 & 0 & \cdots & 0 & 0 \\ 0 & -\alpha_3 & \alpha_{33} & \alpha_4 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots -\alpha_{n-1} & \alpha_{n-1\,n-1} & \alpha_n \\ 0 & 0 & 0 & 0 & \cdots 0 & -\alpha_n & \alpha_{nn} \end{bmatrix} \tag{11}$$

$$B = \begin{bmatrix}\beta_1 & \beta_2 & \ldots & \beta_n\end{bmatrix}^T;\ C = \begin{bmatrix}\gamma_1 & \gamma_2 & \ldots & \gamma_n\end{bmatrix} \tag{12}$$

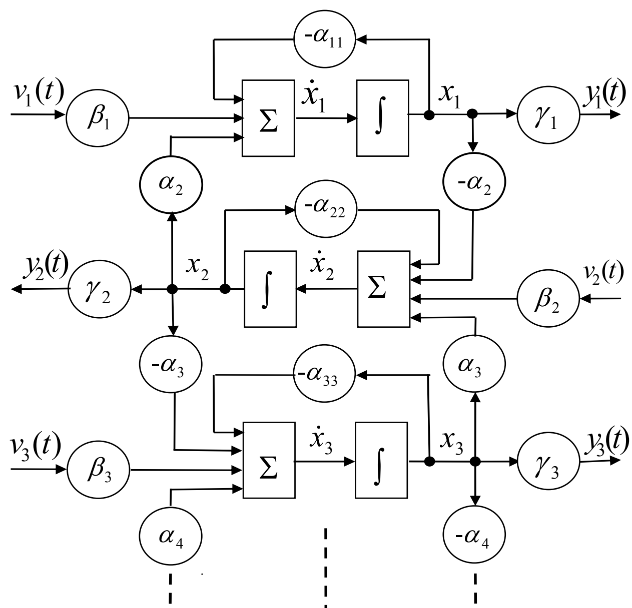

Fig. 2. Physicaly correct structure for generalized Tellegen theorem (α_i should be function of x_i or time)

For the system described by previous equations and structure according Fig. 2 the generalized Tellegen theorem is given by

$$\left\langle x^T(t), \frac{dx(t)}{dt}\right\rangle = 0 \tag{13}$$

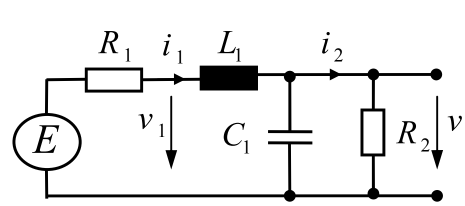

Fig. 3 *RLC* circuit used as linear example

3. LINEAR SYSTEM

In this part the linear example is presented. Let us have a 2nd order *RLC* circuit shown in Fig. 3, which can be described by equations