Available online at www.sciencedirect.com

ScienceDirect

IFAC-PapersOnLine 49-25 (2016) 080–085

Models of Modern Active Devices for Effective and Always Cancelation-free Symbolic Analysis ⋆

Sławomir Lasota *

* *Institute of Electronics, Silesian University of Technology, Gliwice, Poland (e-mail: slasota@polsl.pl).*

Abstract: Generally, there are two reasons for using the symbolic analysis method: to accelerate and improve the accuracy of numerical computations, and to facilitate understanding the dependency of a transfer function vs. some physical parameters. Exact formula tends to be very large and out of the human perception. Thus, in the second approach one expects rather simplified results, but close to the exact one. One of the methods of simplification can be modeling of active devices with some pathological components like nullators, norators, and current and voltage mirrors. In recent years, there were several papers published on this subject. Unfortunately, some of published models are wrong. Seemingly, they work in the proper way, but in practice they model a little bit different device, sometimes non-existing one. Additionally, a few models cannot work at all, because they always yield the singularity. Even disregarding the wrong models, the others still contain auxiliary unit resistors that cause models redundant and not effective enough. They can yield cancelations. The paper presents a formalized method of creating of alternative models, which are free off the drawbacks listed above.

Keywords: design automation, analog circuits, symbolic manipulation, pathological elements, decision diagrams, hierarchical decomposition, structural regularity extraction

1. INTRODUCTION

The symbolic analysis derives the analytical characterization of a circuit behavior in the terms of circuit parameters. Symbolic simulation results are more instructive to designers than numerical ones. However, the number of symbolic terms in the circuit function increases in the exponential way with the number of nodes and branches in a circuit. It is true, if one expects to have the exact results in the form of sum of products. The recent past few years of research in the symbolic analysis has been dedicated to creating of different methods to circumvent this main intrinsic disadvantage. One of a possible "simplification before generation" method is a modeling of active devices with pathological components presented in Fakhfakh et al. (2012) Ch. 3, Sánchez-López (2013). The authors of the idea proves that modeling of active circuits with pathological components dramatically reduce the size of the problem to solve. They refer their conclusion only to the symbolical analysis based on a Modified Nodal Admittance (MNA) matrix. In fact, thanks some tricks with the usage of the pathological components, they reduce analysis to the genuine Nodal Admittance (NA) matrix, even for circuits containing components that are not compatible with NA. Some of these tricks do not work or are useless in other methods, like always cancelation-free 2-graph method. However, in Shi (2015), the author presents application of pathological components to 2-graph method, but he limits himself only to the active component that can be modeled only with pathological components (e.g. ideal op-amp, second-order current conveyors, or presented in the paper dual-X current conveyor). The main advantage of the analysis with the pathological components is that each of them reduces two rows or columns into one in NA matrix. Unfortunately, only a few active devices can be modeled solely with pathological components. For the others, authors tend to use grounded unit resistors. This resistors can generate cancelations. NA analysis method is not itself cancelation-free. However, the direct transfer of these models into the 2-graph method breaks down its main advantage, i.e. being cancelation-free. The paper presents a competitive approach to the models of modern active devices that always retain cancelation-free feature of the 2-graph method. Although, the method is derived from the genuine NA method, it is the topological one, and rather closer to the 2-graph one. Furthermore, their formalism allows us to detect wrong models from Sánchez-López (2013).

The rest of the paper is organized as follows. Some basics of symbolic analysis with hierarchical parameter decision diagrams are presented in Sections II. The idea and some examples of the idealized model of some modern electronic devices creation and the comparison with the competitive solutions are presented in Sections II.

⋆ This research was supported by statutory funds (BK-220/RAu-3/2016) of the Institute of Electronics, Silesian University of Technology granted by Polish Ministry of Science and Higher Eructation No. 8686/E-367/S/2016.

10.1016/j.ifacol.2016.12.014

2. DESCRIPTION OF THE METHOD

2.1 Mathematical Background

Higher Order Summative Cofactors Each linear electric circuit with n nodes plus reference one can be described by equations in a form $\mathbf{I} = \mathbf{Y} \cdot \mathbf{V}$, where $\mathbf{I}$ is a vector of input currents that are forced into nodes $1, \ldots, n$ respectively, $\mathbf{V}$ is a vector of nodal voltages and $\mathbf{V}$ is $n \times n$ admittance matrix. According to the Cramer's rule, j-th nodal voltage $V_j = \frac{\Delta_j^1 \cdot I_1 + \ldots + \Delta_j^n \cdot I_n}{\Delta}$, where $\Delta = det(\mathbf{Y})$ is the determinant of the whole admittance matrix, and $\Delta_j^i = (-1)^{(i+j)} det(\mathbf{Y}_j^i)$ is a 1^{st} order cofactor. $det(\mathbf{Y}_j^i)$ is the determinant of the admittance matrix with i-th row and j-th column removed. According to the superposition rule we can define the current-to-voltage transfer function from node a to node b: $J_{a-b} = \frac{V_b}{I_a} = \frac{\Delta_b^a}{\Delta}$.

The above formula has several drawbacks. First, the current-to-voltage transmittance is not the only and most desirable transfer function. Additionally, the input and output signals need to have the common reference node. Unfortunately, quite often we need to deal with some signals across nodes different from the reference one. It requires postprocessing of results in most symbolic analysis algorithms. Nevertheless, in Sigorskij and Petrenko (1971) the alternative approach was proposed.

Theorem 1. Let Δ_b^a and Δ_b^c are 2 cofactors of the same matrix and they differ in the index of removed rows. Then

$$\Delta_b^a \pm \Delta_b^c = \Delta_b^{(a \mp c)} \tag{1}$$

where row deletion in a form $(a + c)$ means "add row a to row c and then remove a", however $(a - c)$ means "subtract row a from row c and then remove a". A similar relation can be formed for columns. Any deletion in a form $(a \pm b)$ is called the summative one.

According to the Laplace theorem, $det(\mathbf{Y}) = \sum_{c=1}^{n} a_c^i \cdot \Delta_c^i$, where i is some arbitrary chosen row. In the recurrent way each cofactor can be determined by $\Delta_j^i = \sum_{c=1, c \neq j}^{n} a_c^k \cdot \Delta_{j,c}^{i,k}$, where $k \neq j$ is some another arbitrary row. A cofactor with more than one pair of deletions is a higher order cofactor (HOC). In HOC, the sign depends not only on the number of removed rows and columns, but on order of the numbers, as well. To get the proper sign, except the sum of numbers of removed rows and columns, the number of indices exchange to get the increasing order separately for rows and columns should be counted, e.g. $\Delta_{1,3,2}^{3,5,1} = -\Delta_{1,2,3}^{1,3,5}$. If at least one deletion in the higher order cofactor has a summative form, then cofactor will be called a higher order summative cofactor (HOSC). Manipulations with HOSC can be complicated. To simplify operations one cans use the following set of relations:

$$\Delta_{\ldots}^{A,B,\ldots} = -\Delta_{\ldots}^{B,A,\ldots} \tag{2a}$$

$$\Delta_{\ldots}^{(a+b),\ldots} = -\Delta_{\ldots}^{(b+a),\ldots} \tag{2b}$$

$$\Delta_{\ldots}^{(a-b),\ldots} = \Delta_{\ldots}^{(b-a),\ldots} \tag{2c}$$

$$\Delta_{\ldots}^{(a+b),(c+d),(a+c),\ldots} = \Delta_{\ldots}^{(a+b),(c+d),(b+d),\ldots} \tag{2d}$$

$$\Delta_{\ldots}^{(a-b),(c+d),(a+c),\ldots} = -\Delta_{\ldots}^{(a-b),(c+d),(b-d),\ldots} \tag{2e}$$

$$\Delta_{\ldots}^{(a+b),(c-d),(a+c),\ldots} = \Delta_{\ldots}^{(a+b),(c-d),(b-d),\ldots} \tag{2f}$$

$$\Delta_{\ldots}^{(a-b),(c-d),(a+c),\ldots} = -\Delta_{\ldots}^{(a-b),(c-d),(b+d),\ldots} \tag{2g}$$

$$\Delta_{\ldots\ldots\ldots}^{(a+b),\cdots} \pm \Delta_{\ldots\ldots\ldots}^{(c+d),\cdots} = \Delta_{\ldots\ldots\ldots}^{(a\mp c),\cdots} - \Delta_{\ldots\ldots\ldots}^{(b\mp d),\cdots} \tag{2h}$$

$$\Delta_{\ldots}^{(a+a)} \equiv 0 \tag{2i}$$

$$\Delta_{\ldots}^{(a-a)} = 2 \cdot \Delta_{\ldots}^{a} \tag{2j}$$

$$\Delta_{1,2,3,\ldots,n}^{1,2,3,\ldots,n} \equiv 1 \quad \text{even if a matrix is } \boldsymbol{0} \tag{2k}$$

where the ellipsis symbol means the rest of deletions and should be the same in any term. Relations are presented for rows and the similar equations are true for columns, as well. To prove relations (2d)÷(2h) one can apply eq. (1) several times and remove cofactors with multiple deletions for left and right side separately.

Transfer functions HOSC was just created to deal with not grounded signals. Let some multi-input multi-output (MIMO) circuit has two input signals: voltage (forced between node a_u and d_u) and current one (forced between nodes a_i and d_i), and two output signals: voltage drop (between nodes b_u and c_u), and short-circuit current (flowing from node b_i and c_i). From superposition rule four transfer function can be determined to get $U_O = U_i \cdot H_u + I_i \cdot J$, $I_O = U_i \cdot K + I_i \cdot H_i$. Deletions from voltage inputs and current outputs should be added to determine the common denominator: e.g. $\Delta_{(a_u+d_u),(b_i+c_i)}^{(a_u+d_u),(b_i+c_i)}$. To determine some numerator, the input and/or output should be removed from the common denominator, while they are in the set of deletions. Then, a pair $\substack{(I_+ + I_-) \\ (O_+ + O_-)}$ should be added, where I_+, I_-, O_+, O_- are the higher and lower potential of the output or input nodes respectively. Thus, $H_u = \frac{\Delta_{(b_u+c_u),(b_i+c_i)}^{(a_u+d_u),(b_i+c_i)}}{\Delta_{(a_u+d_u),(b_i+c_i)}^{(a_u+d_u),(b_i+c_i)}}$, $J = \frac{\Delta_{(b_u+c_u),(a_u+d_u),(b_i+c_i)}^{(a_i+d_i),(a_u+d_u),(b_i+c_i)}}{\Delta_{(a_u+d_u),(b_i+c_i)}^{(a_u+d_u),(b_i+c_i)}}$, $I = \frac{\Delta_{(b_i+c_i)}^{(a_u+d_u)}}{\Delta_{(a_u+d_u),(b_i+c_i)}^{(a_u+d_u),(b_i+c_i)}}$, $H_i = \frac{\Delta_{(b_i+c_i),(a_u+d_u)}^{(a_i+d_i),(a_u+d_u)}}{\Delta_{(a_u+d_u),(b_i+c_i)}^{(a_u+d_u),(b_i+c_i)}}$. Derivations and details can be found in Lasota (2008, 2012b,a). To find m transfer functions only $m+1$ cofactors should be determined.

Extraction Rules for Elemental Components Let nodes k and l be always a *controlling* nodes, while p and r be a *controlled* ones. k always represents a higher potential. It means that the *controlling voltage* is directed from l to k, but the *controlling current* is a short-circuit current that flows from the node k to l. Output current always flows out from the node r and flows into the node p for the *controlled* nodes. It means that the *controlled current source* or *voltage source* is always directed from p to r. For 2-terminals $p = k = a$ and $r = l = b$. Let Y is some admittance, Z – some impedance, g_m – transconductance of VCCS, A – voltage gain of VCVS, β – current gain of CCCS and r_m – transresistance of CCVS. Then extraction rules are, Lasota (2008, 2012b,a):

$$\Delta_C^R \xrightarrow{\text{extr. } Y} Y \cdot \Delta_{C,(a+b)}^{R,(a+b)} + \Delta_C^R \tag{3a}$$

$$\Delta_C^R \xrightarrow{\text{extr. } Z} Z \cdot \Delta_C^R + \Delta_{C,(a+b)}^{R,(a+b)} \tag{3b}$$

$$\Delta_C^R \xrightarrow{\text{extr. } g_m} g_m \cdot \Delta_{C,(k+l)}^{R,(p+r)} + \Delta_C^R \tag{3c}$$

$$\Delta_C^R \xrightarrow{\text{extr. } A} A \cdot \Delta_{C,(k+l)}^{R,(p+r)} + \Delta_{C,(p+r)}^{R,(p+r)} \tag{3d}$$

$$\Delta_C^R \xrightarrow{\text{extr. } \beta} \beta \cdot \Delta_{C,(k+l)}^{R,(p+r)} + \Delta_{C,(k+l)}^{R,(k+l)} \tag{3e}$$

$$\Delta_C^R \xrightarrow{\text{extr. } r_m} r_m \cdot \Delta_{C,(k+l)}^{R,(p+r)} + \Delta_{C,(p+r),(k+l)}^{R,(p+r),(k+l)} \tag{3f}$$