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Thermostatically controlled appliances in the Home Area Network model

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Abstract: This article presents our work in the field of residential electric energy optimization. We show how to mathematically describe the functionality of thermostatically controlled appliances within the smart home, the end user preferences and other properties. The resulting Mixed Integer Quadratic Problem solution then leads to the optimal schedule of the group of appliances. We further deal with the multiple time-scales control and show several results.

Keywords: Smart Appliances, Smart Building, MIQP, Quadratic Programming, Model Predictive Control

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1 Introduction

The energy consumption plays an important role within all fields of human lives. Although the energy production based on fossil fuel combustion changed greatly the world society, it also caused irreversible changes of the environment on the Earth. People are looking for economically beneficial renewable energy sources for tens of years, but still with no significant success. At present, the diminishing volume of fossil fuels and the increasing pressure to use renewable sources of energy necessitate the integration of such volatile sources into electrical grids. This process, however, results in higher energy costs, and the consumers are thus more willing to change their behavior to either reduce the expenses or maintain them at a reasonable level. One of the relatively few customer-oriented options to optimize energy costs consists in the demand-response principle, which utilizes external information to minimize energy consumption during high price periods. Assuming the constantly changing conditions in electrical grids, and thus also the varying demands, it is vital to provide for automatic optimization excluding the need of user intervention.

Our work presents a method which, after being implemented into the control member, will facilitate the optimal use of appliances and devices within a smart home. As the behavior considered optimal from the perspective of demand-response is often inconsistent with the consumer's requirements for comfortable use of the appliances, the proposed technique offers a compromise through enabling the consumer to select the appropriate strategy. Five universal optimization models are designed within the work; these models facilitate description of common home appliances and local electricity sources. Particular categories are designed based on appliance dynamic behavior throughout modeled time period with respect to the time slot size[1]. Time slot length has been selected as 15 minutes. Appliances that are considered to run only for substantially shorter time period (e.g. electric kettle) are not included in this work. Throughout the existing work one can find studies with wide range of time slot size (e.g. from 1 minute in Widén and Wäckelgard [2010] to 1 hour in Zhu et al. [2012]). After the deeper analysis we found out that studies covering only one group of appliances with bound case studies use shorter time slot sizes (i.e. quid pro quo principle).

The core of the method lies in formulating and optimizing a mixed integer quadratic problem (MIQP). The optimization task yields an operational schedule for the individual appliances, and this scheme considers the energy costs, the working cycle of the appliance, the user demands, the system restrictions and/or other input data.

There are many ways how to categorize appliances based on their dynamic properties. Going throughout recent studies we defined the following groups:

- deferrable appliances
- interruptible appliances
- thermostatically controlled appliances
- distributed generators
- energy storage devices

Modeling of deferrable appliances is described in detail in Bradac et al. [2015], modeling of all above mentioned groups is presented in Kaczmarczyk [2015]. This work is intended to describe modeling thermostatically controlled appliances solely. The aim of these appliances is to keep the desired temperature within the specified room. These appliances must be modeled together with the model of

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[1] Time slot is the least time interval for optimization process – during this interval appliance consumption is considered as constant. It is the least time for switching appliance off between two on cycles or the least time for switching appliance on between two off states.

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the room they are placed. Based on the method of model design, two large groups of studies can be defined:

- *Grey Box* model – design of detailed thermodynamical model of the building. For this purpose one need to know specific details of building construction (kind of used material etc.), building dimensions, particular rooms dimensions and connections between them, windows size etc. Once this information is known, equivalent thermal circuit can be created[2] Based on this circuit, system of differential equations can be easily derived. Grey Box model design is described in Lienhard [2008]. Studies Pisello et al. [2012] and Scotton [2012] use this approach to create mathematical models of building.
- *Black Box* parametric model – this model can be calculated by parametric identification methods (e.g. least square method). For successful identification appropriate sets of measured control actions and room temperatures are necessary. This group of models is not used within this work.

2 Grey Box models and Finite element method

Thermodynamical properties of the complex building cannot be described by one, even complicated, equation. The building must be divided into smaller parts, that correspond to the physical concept. These parts should be described and modeled individually. The basic property of every such part is the thermal capacity and heat transfer between the other parts. Thermal capacity depends on the mass of specific material and its type. Thermal energy between all modeled parts can be transferred by three means - *conduction*, *convection*, *radiation*.

The model created by above mentioned rule can exactly describe all parts of the building, every layer of wall material, every window. Even fundamental parts of the building equipment placed inside can be described that way. Hence the most important part of modeling is to select proper model simplification. Only substantially simplified model can be included into subsequent computations.

2.1 Basic concepts

For creating a model heat transfer through the wall of thickness A two phenomena must be considered - material conduction and convection of thin layer of air adjoining to the wall from each side. The heat transfer coefficient for the wall inner side h_{in} differs from the coefficient for the outer side h_{out}. Thermal resistance for conduction for both wall sides can be computed as $R_i = \frac{1}{h_{in}\cdot A}$, respectively $R_o = \frac{1}{h_o\cdot A}$. The total thermal resistance can be determined as $\frac{1}{h_i\cdot A}+\frac{L}{\lambda\cdot A}+\frac{1}{h_o\cdot A}$ (see picture 1). Thermal capacity element itself is modeled in the geometric center of the wall. Hence the middle part of thermal resistance expression must be split into $\frac{L}{2\cdot\lambda\cdot A}+\frac{L}{2\cdot\lambda\cdot A}$.

Both radiation heat transfer and heat transfer through windows is omitted in this version of thermal model. Windows area is joined with adjoining walls. The resulting equation for thermodynamic equilibrium of the room 1 is then

$$\rho_w V_w c_w \frac{dT_w}{dt} = \frac{T_{out}-T_w}{R_{out}} + \frac{T_1 - T_w}{R_{in}}, \text{ where} \qquad (1)$$

$$R_{out} = \frac{1}{h_{out}\cdot A_w} + \frac{L}{2\cdot\lambda\cdot A}$$

$$R_{in} = \frac{1}{h_{in}\cdot A_w} + \frac{L}{2\cdot\lambda\cdot A}$$

In case of the inner wall the element T_{out} in the equation should be replaced by the the opposite room temperature T_2. Instead of the outer thermal resistance R_{out} the value of inner thermal resistance R_{in} is applied.

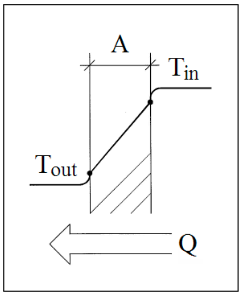

Fig. 1. Wall heat transfer – conduction through wall mass and adjoining air layer convection.

2.1.1 Room thermal capacity The basic assumption when modeling room thermal capacity is that the temperature throughout the room is homogenous (and labeled as T_r). Thermal capacity of the air inside the room is $C_r = \rho_r V_r c_r$ and is proportional to the air density, volume and heat capacity. For each wall the heat leakage $R_{w,r}$ must be computed according to 1 ($w \in \mathbb{W}$ - set of surrounding walls). Heating or cooling the room is realized by driving the hot or cold air of mass m and temperature T_{in} from the heating/cooling unit. The temperature of the air entering back into the unit equals to the room air temperature. In case there are any other thermal sources in the room (electronic devices), their influence may be considered within the element $\dot{Q}_{int}$. The room temperature is then described by equation

$$\rho_r V_r c_r \frac{\mathrm{d}T_r}{\mathrm{d}t} = \sum_{w\in\mathbb{W}} \frac{T_w - T_r}{R_{w,r}} + \dot{m}_{in} c_{in}(T_{in} - T_r) + \dot{Q}_{int}, \qquad (2)$$

2.2 Mathematical model

As stated above, basic parameters of the building construction may help to build the mathematical model - the system of non-linear differential equations, which, in matrix form, can be written as

$$\dot{\boldsymbol{x}} = \mathbf{A}\boldsymbol{x} + f(x,u) + \boldsymbol{d}(t) \qquad (3)$$
$$\boldsymbol{y} = \mathbf{C}\boldsymbol{x},$$

where $f(x,u)$ is non-linearity in the form $(vstup \cdot stav)$ and $\boldsymbol{d}(t)$ is a vector of time-dependent disturbances influencing the system (present version of the model takes the influence of the outer temperature as measurable disturbance).

2.2.1 Newton's linearization method The large group of models of physical systems (including the model created in our work) are non-linear. Unfortunately, non-linear system

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[2] Equivalent thermal circuit describes thermal elements by their analogical electronic elements.