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Optimal Predictive Control - A brief review of theory and practice

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Abstract: Automatic control plays important role in both industrial and commercial engineering applications. Many of these applications use predictive control strategies implemented on digital computers to control dynamic systems. This paper aims to present methods used to design predictive control with the knowledge of the controlled system‘s model.

Keywords: Control, Optimization, Predictive Control, Distributed Computer Control Systems, Simulation

1. INTRODUCTION

In the *classical control* the design is carried out using transformation of the dynamic system model into *frequency domain* or *s-plane*. Thanks to the aforementioned transformation, this approach is primarily suitable for *linear time-invariant systems*. An exact description of the internal system dynamics is not necessary for classical design, only input/output behavior is considered Lewis (1992). In Fig. 1 is shown typical closed-loop diagram of feedback control designed in the *s-plane*. Design can be carried out using graphical methods and often requires engineer's intuition to fine-tune the resulting compensator, which usually is simple in structure (e.g. PID compensator). The design of the compensator can be done in terms of open loop properties which are easily mapped to closed-loop properties due to the structural simplicity of the designed compensator Lewis (1992).

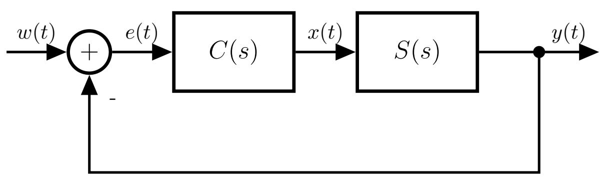

Fig. 1. Classical control loop

Advantage of the classical control strategies is robustness of designed control systems, which can yield good closed-loop performance even in spite of measurement noise and uncertainties in the mathematical model which was used to design them. Robust designs are carried out using notions like gain and phase margin. On the other hand classical control theory is difficult to apply on MIMO and large scale systems Lewis (1992).

Nowadays, it is common for complex control problems (such as the one shown in Ozana et al. (2012)) to be solved using so called *modern control theory*. Modern control designs are carried out almost exclusively in time domain. These approaches utilize a *state-space model*, often called *plant*. Linear time-invariant continuous-time model can be in state-space mathematically described by a first-order vector differential equation of the form

$$\begin{aligned} \dot{x}_c(t) &= A_c x_c(t) + B_c u_c(t) \\ y_c(t) &= C x_c(t) + D u_c(t) \end{aligned} \tag{1}$$

where $x_c(t) \in \mathbb{R}^n$ is a vector of continuous-time internal states, $u_c(t) \in \mathbb{R}^m$ is a vector of continuous-time control inputs, $y_c(t) \in \mathbb{R}^p$ is a vector of continuous-time measured outputs, and $A_c \in \mathbb{R}^{n\times n}$, $B_c \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$, $D \in \mathbb{R}^{p\times m}$ are matrices whose elements specify the dynamic properties and behavior of the continuous-time plant model Lewis (1992).

The main aim of this paper is to present a review of the literature concerning model predictive control and its application. In the experimental part, an example of a physical plant model, a design and application of the predictive optimal control are presented.

2. THEORETICAL PART

Due to the fact, that modern control is often implemented using computers, which natively operate in discrete-time, state-space mathematical models (2) comprised of first order vector difference equations are used.

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned} \tag{2}$$

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where $k \in \mathbb{Z}^+$ is number of the sample and corresponds to the time kT_S, T_S is a sampling period, $x(k) \in \mathbb{R}^n$ is a vector of discrete-time internal states, $u(k) \in \mathbb{R}^m$ is a vector of discrete-time control inputs, $y(k) \in \mathbb{R}^p$ is a vector of discrete-time measured outputs, and $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$, $D \in \mathbb{R}^{p\times m}$ are matrices whose elements specify the static and dynamic properties and behavior of the discrete-time plant model. Discrete-time dynamic systems describe either (I) inherently discrete systems, e.g. bank saving account balance at the k-th month, (II) Transformed continuous-time systems, while the latter is much more common case.

2.1 Model Predictive Control

The control strategy known as *Model Predictive Control* (MPC) or *Receding Horizon Control* (RHC) and *Moving Horizon Optimal Control* is a modern approach in control and aims at transformation of the control problem into an optimization one Scattolini (2009), which enables the optimization of the the controlled plant behavior over controllable plant inputs $u(k)$ and predicted evolution of the plant state $\hat{x}(k+i|k)$.

Prediction of the plant state is obtained by utilizing a explicit numerical model of the controlled plat. Therefore the model is the essential element of an MPC controller. In the real world, the model is always imperfect estimation of the physical plant, thus the plant state forecast is never completely accurate. This inaccuracy can be partially overcome by implementing feedback from the output of the plant. Rawlings (2000).

Time domain, input/output, step or impulse models were usually used for the early industrial applications of the MPC controllers Richalet et al. (1978); Cutler and Ramaker (1979); Prett and Gillette (1980). Later it has become more common for researchers to use linear models in the state-space form (2) Morari and Lee (1999); Bemporad and Morari (1999); Froisy (2006); Qin and Badgwell (2003); Farina et al. (2013); Borrelli et al. (2015). Replacing input/output models by state-space models in MPC controller shows no disadvantage, contrary it provides several advantages, including easy generalization to multi-variable systems, ease of on-line computation, etc. Rawlings (2000).

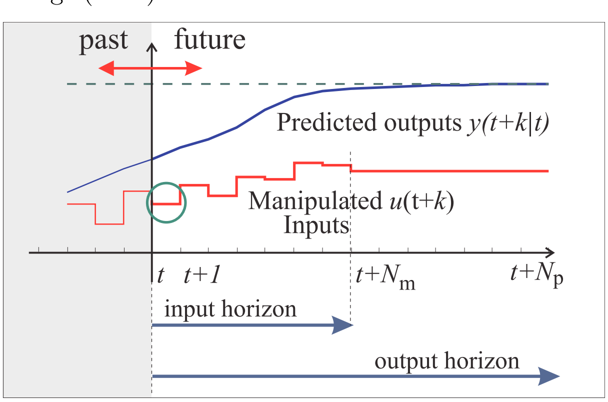

Fig. 2. Receding horizon strategy Bemporad and Morari (1999)

A MPC controller is implemented by solving optimization program, such as (3). The result of the optimization (4) is then applied according the *receding horizon* philosophy: At the instant k only the first optimal control output $u(k|k)$ is actually applied to the plant. The remaining optimal control outputs are discarded and a new optimal control program is solved at the instant $k+1$. This is illustrated in Fig. 2 and described in Algorithm 1.

$$\begin{cases} \min_{u_{\cdot|k}} J(k, N_p, N_m, x(k), \hat{x}_{\cdot|k}, u_{\cdot|k}) \\ \text{subject to} \\ \hat{x}(k+1) = A\hat{x}(k) + Bu(k) \\ G_1 u_{\cdot|k} \leq g_1 \\ G_2 \hat{x}_{\cdot|k} \leq g_2 \\ \text{“stability constraints”} \end{cases} \tag{3}$$

In (3) is by $J(k, N_p, N_m, x(k), \hat{x}_{\cdot|k}, u_{\cdot|k})$ denoted a cost function, $\hat{x}_{\cdot|k}$ and $u_{\cdot|k}$ denote sequences of the predicted plant states and control outputs respectively. The k after the "|" sign denotes the instant at which these sequences were calculated.

$$\hat{x}_{\cdot|k} = \big(\hat{x}(k+1|k), \hat{x}(k+2|k), \ldots, \hat{x}(k+N_p|k)\big) \tag{4a}$$
$$u_{\cdot|k} = \big(u(k|k), u(k+1|k), \ldots, u(k+N_m-1|k)\big) \tag{4b}$$

N_p denotes the length of the *prediction horizon* or *output horizon* and N_m denotes the length of the *control horizon* or *input horizon* ($N_m \leq N_p$). When $N_p = \infty$, we refer to this as the *infinite horizon problem*, and similarly, when N_p is finite as the *finite horizon problem* Bemporad and Morari (1999). The last three constraints in (3) are not essential for the MPC in its standard form, but they will act upon optimization process when designing a MPC controller for the real plant, since they arise from the properties of the real plant and stability requirements.

The cost function $J(k, N_p, N_m, x(k), \hat{x}_{\cdot|k}, u_{\cdot|k})$ may have different forms depending on the optimization program type (linear program, quadratic program, etc.). A receding horizon implementation typically utilizes quadratic cost function of the form (5) which is used to solve open-loop optimization program (3) Bemporad and Morari (1999).

$$\begin{aligned} &J(k, N_p, N_m, x(k), \hat{x}_{\cdot|k}, u_{\cdot|k}) = \\ &\hat{x}^T(k+N_p|k) Q_0 \hat{x}(k+N_p|k) + \\ &\sum_{i=0}^{N_p-1} \hat{x}^T(k+i|k) Q \hat{x}(k+i|k) + \\ &\sum_{i=0}^{N_m-1} u^T(k+i|k) R u(k+i|k) \end{aligned} \tag{5}$$

A basic MPC law is described by the following algorithm:

2.2 Issues with Model Predictive Control

Computational Complexity The computational complexity of the solver for the optimization program (3) is often of great concern. It depends on the internal workings of the solver, choice of the performance index (linear, quadratic