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Optimal Scheduling of Vaccination Campaigns Using a Direct Dynamic Optimization Method

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Abstract: The scheduling of vaccination campaigns determines the feasibility and effectiveness of controlling the proliferation of an epidemic disease. Optimal vaccination strategies can be derived by using optimization methods. However, most of the solution approaches for this target have been based on the indirect dynamic optimization methods, leading to limitations in the scheduling of vaccination campaigns. In this study, the scheduling problem is solved by an efficient direct method known as the combined shooting and collocation approach. In this way, both control and state constraints can be efficiently treated. More importantly, this method allows solving the problem with multiple impulsive vaccinations at arbitrary time points. As a result, the quality of the vaccination can be essentially improved in comparison to the results from the literature.

Keywords: SIR dynamic model, Vaccination campaigns, Optimal control problem, Dynamic optimization, Multiple-shooting, Orthogonal collocation.

1. INTRODUCTION

In recent years, the interest in studying the dynamic behavior and control of disease proliferation has been growing, leading to a research field known as mathematical epidemiology. One of the most important applications of mathematical epidemiology is the analysis of the effectiveness of interventions in public health when dealing with emergent epidemic and infectious diseases. Since this kind of diseases represent a threat to public health security, an emergency response plan is required. Thus, one key step is the deployment of efficient vaccination campaigns to control and, possibly, eradicate the disease. However, in practice, a variety of factors (e.g. costs, the number of population, periods of vaccination, etc.) have to be taken into account in planning such vaccination campaigns, making it a challenging task.

Model-based optimization methods integrate these factors into an optimal control problem which solution provides an optimal scheduling of vaccination strategies. A susceptible-infected-recovered (SIR) dynamic model is commonly used to represent the behaviors of the population in presence of an epidemic disease (Kermack and McKendrick, 1927; Hethcote, 1989), based on which optimization problems have been formulated and solved by different approaches in previous studies.

The existing approaches to nonlinear optimal control can be classified into indirect and direct methods (Von Stryk and Bulirsch, 1992). In the *indirect* methods, the solution is derived based on the optimality conditions (Bryson, 1975), leading to a two-point-boundary problem. It is known that inequality constraints on state variables are difficult to be addressed by indirect methods (Bertsekas, 1995). On the other hand, the *direct* methods use a discretization scheme to transform the optimal control problem into a nonlinear program (NLP) problem which can be efficiently solved by an NLP solver. In this way, state constraints can be easily treated (Diehl et al., 2006).

Investigated by Verriest et al. (2005), both *constant* and *impulsive* vaccination strategies were employed as control profiles. The scheduling problem was formulated as an unconstrained optimal control problem and solved by the method of dynamic programming. Solutions with up to three vaccination campaigns with predefined time points were presented. The mathematical analysis of the impulsive vaccination strategy has been presented in (Liu et al., 2012) and more recently in (Laguzet and Turinici, 2015).

Using a SIR dynamic model with a varying size of population, Yusuf and Benyah (2012) defined a continuous immunization rate as a bounded control variable. The Pontryagin's Maximum Principle was used to derive an analytical solution of the problem. Similarly, Laarabi et al. (2013) considered a continuous vaccination approach to solve the optimal control problem in which a SIR dynamic model with saturated incidence rates was used. The solution was also obtained by the Pontryagin's Maximum Principle. Similar studies can be found in (Kar and Jana, 2013; Bakare et al., 2014; Joshi et al., 2015; Nowzari et al., 2016). Obviously, the *continuous* vaccination strategy from these studies is not realistic for application.

Da Cruz et al. (2009) presented a multiple impulsive vaccination approach and proposed to use multi-objective minimization to address the vaccination scheduling problem

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where both control and state constraints were considered. Due to the high complexity of the resulting optimization problem, a genetic algorithm was employed. However, it is well-know that the methods of stochastic search are in general inefficient.

A framework for parameter estimation of disease models based on direct methods was presented in Word et al. (2012). This work used the collocation on finite element method to solve the estimation problem considering a fixed step size. They showed that this direct method provides a more efficient solution approach and is scalable to large spatially distributed estimations. Moreover, the direct methods are not only limited to solve estimation problems in epidemics, but also those problems related to control strategies like the scheduling of vaccination campaigns.

In this study, we propose to use a state-of-the-art direct dynamic optimization method, namely the combined multiple shooting with collocation approach, for solving the optimal scheduling problem. This leads to three new contributions in comparison to the previous studies. First, both control and state constraints, which are indeed necessary in many situations, can be explicitly introduced into the problem formulation to facilitate the epidemic campaign. Second, multiple impulsive vaccinations desired in the practice can be implemented as control strategy. Third, the time points (instants) of multiple impulsive vaccinations can be readily optimized, which, in comparison to fixed time instants, allows different vaccination levels and provides the optimal health policy actions according to the constraints of the problem and the desired behaviour, as will shall show. Computation results demonstrate the advantages of the proposed approach by essentially improving the quality of the epidemic campaign. Moreover, the method presented in this paper is general and can be applied in other disease campaign problems.

2. THE SIR EPIDEMIC MODEL

Commonly used in mathematical epidemiology and theoretical biology, the susceptible-infected-recovered (SIR) model is a simplified epidemiological model that describes the transmission of a disease through the population (Anderson et al., 1992). This was extended from the susceptible-infected model (Hethcote, 1989) which is based on the prey-predator model widely used in different areas. It is called the SIR model because it classifies the population in the following categories:

- *Susceptible population* (S) - describing the individuals who are not infected yet but may become;
- *Infected population* (I) - describing the individuals who are infected and can transmit the disease to the susceptible ones;
- *Recovered population* (R) - describing the individuals who have been recovered from the disease or vaccinated before infected.

For an initial population of P individuals and considering that it keeps constant during the vaccination period by balancing the population mortality and birth rates, it leads to the SIR dynamic model as follows (Anderson et al., 1992)

$$\begin{aligned} \dot{S}(t) &= \mu P - \mu S(t) - \frac{\beta}{P} I(t)S(t)\,, \\ \dot{I}(t) &= \frac{\beta}{P} I(t)S(t) - \gamma I(t) - \mu I(t)\,, \\ \dot{R}(t) &= \gamma I(t) - \mu R(t)\,, \end{aligned} \tag{1}$$

where β is the disease transmission rate from infected to susceptible individuals, γ is the recovery rate of infected individuals and μ is the rate of new susceptible people. Since the population is constant along the time, the following inherent relationship holds for every time instant

$$P = S(t) + I(t) + R(t)\,. \tag{2}$$

Fig. 1 shows a typical free-response of the SIR dynamic model, i.e., the behavior of the system when no preventive methods are applied to control the proliferation of the disease. Here, we consider a population of $P = 1000$ individuals with initial susceptible and infected populations of $S(0) = 0.9P$ and $I(0) = 0.1P$. The values of the rate parameters are $\mu = 0.0214$, $\beta = 2.700$, $\gamma = 0.1633$ (Da Cruz et al., 2009).

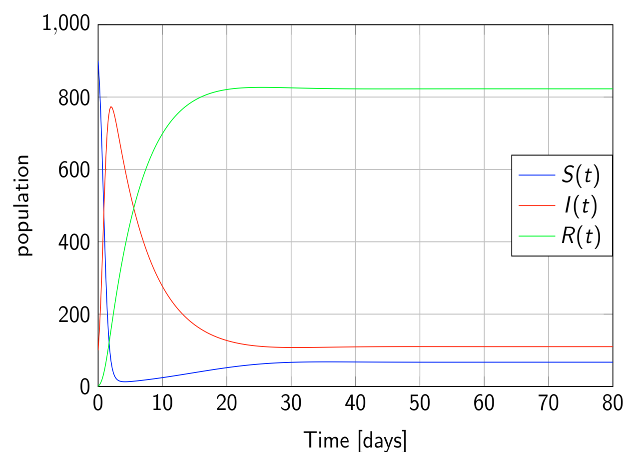

Fig. 1. Free response of the SIR dynamic model.

As can be seen, the number of infected people $I(t)$ reaches a peak value of almost 80% of the total population and reduces to a steady-state value of around 12% of P, which is a considerable quantity. To minimize the infected individuals, it is necessary to develop an efficient treatment strategy that controls the proliferation of the disease and, if possible, finally eradicates it. One of the most popular preventive strategies for controlling epidemic diseases is the vaccination of susceptible population, which is carried out through scheduled vaccination campaigns. In the next section, we will present two different strategies commonly used to achieve this target.

3. VACCINATION STRATEGIES

The vaccination is a very effective strategy which prevents the population from being infected. To consider the effect of the vaccination in the SIR dynamic model, either continuous (Pei et al., 2008; Yusuf and Benyah, 2012; Laarabi et al., 2013; Nowzari et al., 2016) or impulsive (Verriest et al., 2005; Da Cruz et al., 2009; Liu et al., 2012) vaccination has been proposed in previous studies. In the form of continuous vaccination, the population is vaccinated at a