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IFAC-PapersOnLine 49-20 (2016) 001–006

State and parameter identifiability of a non-linear grinding mill circuit model

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Abstract: The states and unknown parameters of a non-linear model of a grinding mill circuit are shown to be identifiable from manipulated, controlled and measurable variables. All the states in the mill and sump, the breakage rates in the mill, the mill and cyclone parameters and the mill power draw parameters can be algebraically expressed in terms of known variables. Theoretically, the tiered approach to estimate all states and parameters enables model-fitting without the need for unnecessary assumptions regarding model parameters. The analysis shows the necessity of accurate density and flow-rate measurements across the circuit, and especially the cyclone, to estimate all states and parameters. The aim is to enable non-linear model-based control through model characterisation using available real-time plant measurements.

Keywords: comminution, grinding mill, identifiability, parameter estimation, state estimation

1. INTRODUCTION

The general control objectives for a grinding mill circuit are to produce the maximum possible quantity of a product at a specified quality while maintaining a stable process, decreasing power usage and reducing grinding media. The main challenges are to manage strong variable coupling, unmeasured disturbances, variation of parameters over time, process non-linearities and large time delays (Wei and Craig, 2009). Because of the increased economic pressures on mineral processing operations, controllers are required to maintain variables within a tighter range while pushing the process towards the optimal operating region. This optimal region generally lies at the limit of the milling circuit capacity.

Controllers developed on linear step-test models assume process dynamics remain constant throughout continual operation. Given the variability in ore characteristics the linear controllers do not account for the movement in operating conditions. Although non-linear model-based controllers are better suited to capture the non-linear movement in operation conditions (Coetzee et al., 2010), the non-linear models have states and parameters which are not readily observable from available plant measurements. Sufficient and accurate measurement instrumentation is required for adequate characterisation of the operating conditions which enables improved control and operation closer to the optimum operating point.

The non-linear phenomenological population balance model of le Roux et al. (2013) can be used to describe the grinding mill circuit shown in Fig. 1. The approach in the derivation of this model was to use as few fitted parameters as possible for a reasonably accurate model with responses in the correct directions. The main purpose of the model is for use in non-linear model-based control strategies of grinding mill circuits. However, there is not yet a procedure to fit model parameters to dynamic industrial data.

This paper aims to shows that all the model states and parameters are identifiable. Using a tiered approach where parameters and states are determined in a specific order, the states and parameters are expressed as algebraic equations in terms of previously calculated variables as well as controlled, manipulated and measured variables. Only real-time measurable variables commonly available on industrial circuits are used of the analysis.

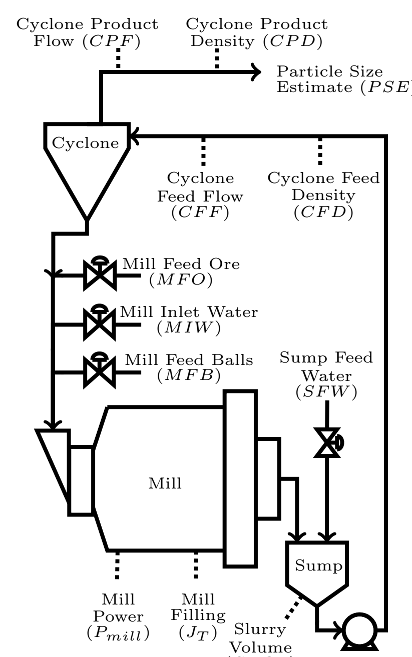

Fig. 1. A single-stage closed grinding mill circuit.

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Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2016.10.087

2. OBSERVABILITY AND IDENTIFIABILITY

Identifiability is the ability to express system parameters in terms of measured data, where measured data refers to measured, manipulated and controlled variables and their time derivatives. Because the parameters of a system can be considered as state variables with time-derivatives of zero, identifiability can be seen as a special case of observability. For this article the definition of algebraic identifiability is understood as in Xia and Moog (2003).

3. GRINDING MILL CIRCUIT PROCESS

3.1 Process Description

The three main elements of the single-stage closed grinding mill circuit in Fig. 1 are a semi-autogenous (SAG) mill, a sump and a cyclone. Four streams enter the mill: mined ore, water, steel balls and underflow from the cyclone. The ground ore in the mill mixes with water to create a slurry. The slurry in the mill is discharged through an end-discharge-grate where the aperture size of the end-discharge-grate limits the particle size of the discharged slurry. The discharged slurry is collected in a sump where it is diluted with water before it is pumped to the cyclone. The cyclone, responsible for classification, separates the in- and out-of-specification sized ore discharged from the sump. The in-specification sized particles of the slurry pass to the overflow of the cyclone, while the out-of-specification particles pass to the underflow. The underflow is passed back to the mill for the out-of-specification particles to be ground further. The overflow contains the final product which is passed to downstream processes (Napier-Munn et al., 1999).

3.2 Manipulated, Controlled and Measured Variables

The manipulated, controlled and measured variables of the circuit in Fig. 1 are described in Table 1. As seen in the survey by (Wei and Craig, 2009), the manipulated and controlled variables in Table 1 are measurable in real-time and are commonly available on industrial circuits. The mill power (P_{mill}) and cyclone feed density (CFD) are variables commonly measured on industrial circuits, whereas cyclone product density (CPD) and mill feed particle size distribution (α_r and α_f) are less commonly measured. The survey does not include cyclone product flow-rate (CPF) as a measured variable for any of the plants surveyed.

A possible reason why CPD and CPF are not common measurements is because these measurements are not seen as important for plant operation. Although a false assumption, operators generally assume CPD stays constant and can be determined through hand-drawn samples. Another reason why CPF and CPD are not common measurements is because the amount of air in the cyclone overflow pipe influence instrumentation accuracy. A possible solution is to pass the cyclone overflow through a U-shaped pipe to create an airless flowing volume of slurry. As shown in the analysis in Section 4, both dynamic variables are necessary to calculate model states and parameters. For the purposes of this study it assumed both measurements are available.

Table 1. Description of circuit variables

Manipulated Variables	
MIW	flow-rate of water to the mill [m^3/h]
MFO	feed-rate of ore to the mill [t/h]
MFB	feed-rate of steel balls to the mill [t/h]
SFW	flow-rate of water to the sump [m^3/h]
CFF	flow-rate of slurry to the classifier [m^3/h]
Controlled Variables	
$LOAD$	volume of mill volume filled with slurry [m^3]
$SVOL$	volume of slurry in sump [m^3]
PSE	product particle size fraction $<$ 75 micron [-]
Measured Variables	
α_f	Fraction fines in the ore [-]
α_r	Fraction rock in the ore [-]
CFD	cyclone feed density [t/m^3]
CPD	cyclone product density [t/m^3]
CPF	cyclone product flow-rate [m^3/h]
P_{mill}	mill power draw [kW]

Table 2. Nomenclature

Known Parameters	
α_P	Fractional power reduction per fractional reduction from maximum mill speed
α_{ϕ_f}	Fractional change in kW/fines produced per change in fractional filling of mill
α_{speed}	Fraction of critical mill speed
C_1	Constant
C_2	Constant
D_B	Density of steel balls [t/m^3]
D_S	Density of feed ore [t/m^3]
D_W	Density of water [t/m^3]
ε_{sv}	Max fraction solids by volume of slurry at 0 slurry flow
P_{max}	Maximum mill motor power draw [kW]
v_{mill}	Mill volume [m^3]
Unknown Parameters	
α_{su}	Parameter related to fraction solids in underflow
C_3	Constant related to fraction solids in cyclone inflow
C_4	Constant related to fraction fines in cyclone feed solids
δ_{P_s}	Power-change parameter for fraction solids in the mill
δ_{P_v}	Power-change parameter for volume of mill filled
ε_c	Parameter related to coarse split [m^3/h]
ϕ_b	Steel abrasion factor [kWh/t]
ϕ_f	Power needed per tonne of fines produced [kWh/t]
ϕ_r	Rock abrasion factor [kWh/t]
$\varphi_{P_{max}}$	Rheology factor for max mill power draw
$v_{P_{max}}$	Fraction of mill volume filled for max power draw
V_V	Volumetric flow per flowing volume driving force [1/h]
χ_P	Cross-term for maximum power draw

3.3 Model Description

A brief overview of the model in le Roux et al. (2013) is given here. The nomenclature for the model is shown in Table 2. The model divides the circuit into four modules: a feeder, a SAG mill with an end-discharge-grate, a sump and a cyclone. For the model equations, V denotes a flow-rate in m^3/h and X denotes the states of the model as volumes in m^3. Table 3 provides a description of the subscripts for V and X. The first subscript indicates the module considered (feeder, mill, sump or cyclone), the second subscript specifies which of the six states are considered (rocks, solids, coarse, fines, balls, water), and in the case of flow-rates the final subscript indicates an inflow, outflow or underflow.

The model uses six states to represent the constituents of the charge in the milling circuit. The six states are rocks,