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Entropy-based estimation of bubble size distributions in froth flotation using B-spline functions ⋆

Jie Han, Chunhua Yang, Xiaojun Zhou, Weihua Gui

School of Information Science and Engineering, Central South University, Hunan 410083, China (e-mail: michael.x.zhou@csu.edu.cn).

Abstract: Bubble size distribution is an important feature in froth flotation processes and the B-spline estimation is a common way to approximate the bubble size distribution by using the least square algorithm. However, it is difficult to determine the number of basis functions of B-spline. In this study, the analysis based on the unbiased cross entropy is proposed to evaluate the estimated models. Experimental results have shown the feasibility and effectiveness of the proposed method.

Keywords: Froth flotation process, bubble size, probability density function, B-spline

1. INTRODUCTION

Froth flotation is a separation process aimed to extract valuable minerals from raw ore. As a multivariate process, froth flotation is influenced by multiple factors such as impeller speed, reagent addition and inlet air flow (Yang et al. [2009a]). Since the nature of underlying microscopic phenomena is chaotic and complicated, the modelling and control of flotation processes are difficult (Bao et al. [2008]). Experiences have shown that the efficiency of flotation process is highly correlated to the froth characteristics. In Moolman et al. [1996], the control of flotation processes is heavily dependent on the experience of human operators by observing the visual appearance of the surface froth, which varies with each individual and lacks objective criterion. However, with the advancement in digital image processing techniques, quantification of froth properties becomes particularly convenient.

As one of the dominant visual features, bubble size can provide important information about flotation process status (Feng. [2000]). For bauxite flotation, froth bubble size is not only related to technical indices such as concentrate grade and tailings grade, but also connected with technological parameters like pulp acidity and pulp density. In order to get generic relationships for the performance of flotation froths, Wang et al. [2009] proposed an empirical formula about bubble size distributions. It is worth noticing that the probability density function (PDF) of surface bubble size is found to be non-Gaussian, and Yang et al. [2009b] used nonparametric estimation technique to approximate this distribution. Based on the theory of stochastic distribution control, the output PDFs can be tracked to a target distribution shape by using control approaches (Guo et al. [2008]). Considering the relationship of bubble size distribution and the process of froth flotation, Zhu et al. [2014] proposed a novel reagent dosage predictive control method which was based on the PDF of bubble size.

Since the estimation of bubble size distribution is a prerequisite step of tracking a target distribution shape, a method using B-spline function and entropy to approximate the PDF of bubble size is proposed in this paper. This method includes three steps. First, the PDF of bubble size distribution is described by using kernel density estimators. Second, we use a linear combination of B-splines to approximate the kernel density estimation by least-squares method. Final step, the best model of linear combination is determined by entropy analysis. The proposed method can describe the bubble size distribution effectively by the weights of B-spline estimation, and it can be applied to the further control of flotation process.

The remainder of the paper is organized as follows. The mechanism of the gold-antimony flotation process and the approach of characterizing the bubble size are described in the section 2. Section 3 presents the B-spline estimator based on entropy and experimental results is discussed in Section4. Section 5 provides the conclusion of this paper.

2. PROCESS DESCRIPTION

2.1 Gold-antimony flotation process

The process of froth flotation makes use of the physicochemical properties of mineral grains to concentrate valuable materials. After grinding the raw ore to a fine size and mixing them with water, frothing reagents and collecting reagents, the pulp is fed in the flotation cells. The sparger at the bottom of the container can continuously inject air into the pulp to form a large number of bubbles (see Fig. 1). Chemical reagents can adjust the surface

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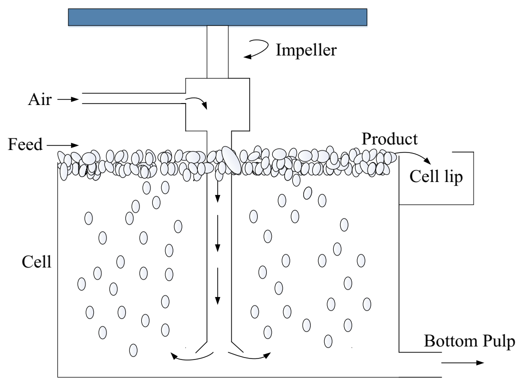

Fig. 1. The process of froth flotation

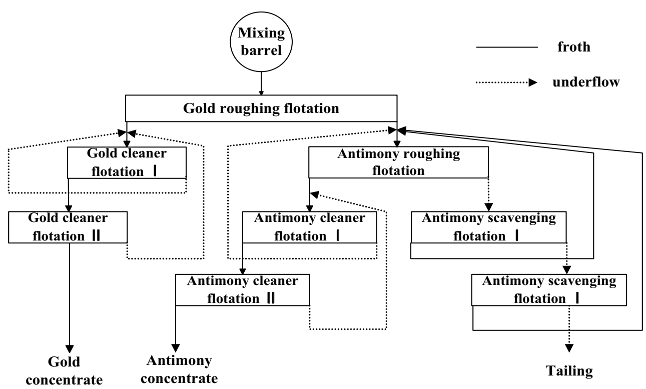

Fig. 2. Process flow of gold-antimony flotation

hydrophobicity of the fine particles. In general, the valuable material particles are hydrophobic and adhered to the bubble surface. Due to the flotage effect, air bubbles float to the top and can be used to collect concentrate product. The hydrophilic particles such as tailings settle to the bottom and are recycled for further treatment.

In this paper, the gold-antimony flotation is mainly studied and Fig. 2 shows its process flow (Wu et al. [2015]). The mineral particles containing gold are first selected by the gold roughing flotation and gold cleaner flotation I, II, the mineral particles containing antimony are then collected by the antimony roughing flotation and antimony cleaner flotation I, II. Scavenging flotation process is used to improve the recovery rate of minerals.

2.2 Bubble size feature extraction

Image measurement set-up for the flotation process always consists of a RGB camera and a lamp above the surface of the froth layer. As discovered, froth images collected from industry field show that each bubble has only one total reflectance point, and this property enables bubble segmentation to be much easier.

Although there are many methods about froth image segmentation, including valley-edge detection and tracing technique (Wang et al. [2003]), white spots detection (Wang et al. [2005]), watershed transformation (Bartolacci et al. [2001]) and wavelet approach (Liu et al. [2005]), they all have poor effect when dealing with the froth image

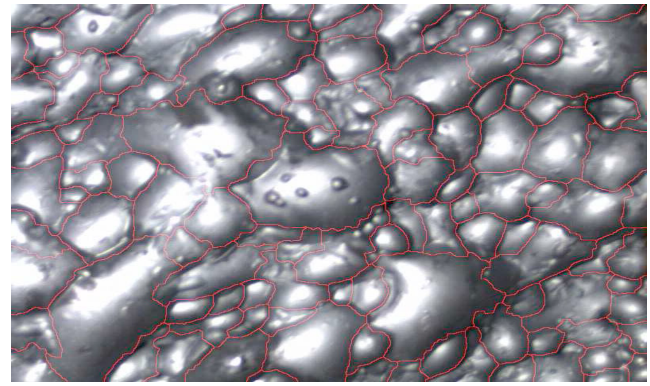

Fig. 3. The segmentation of froth image

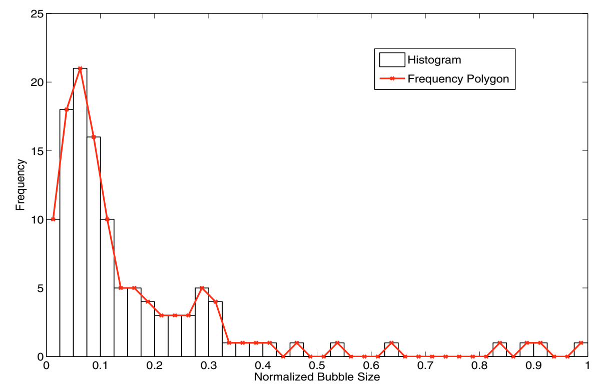

Fig. 4. The histogram and frequency polygon of bubble size distribution

with both large and tiny bubbles. In this paper, a two-pass watershed algorithm explored by Gordon Forbes (Forbes, G. [2007]) is adopted to segment image. Before using this algorithm, gray-scale dilations-based morphological reconstruction is applied to denoising. Fig. 3 shows the segmentation result of antimony roughing flotation image and Fig. 4 is the relevant histogram and frequency polygon of bubble size distribution.

3. B-SPLINE ESTIMATION BASED ON ENTROPY

3.1 B-spline estimation

There are many ways to estimate the irregular distribution of some random variables, especially histogram, frequency polygon, shift average histogram, kernel methods and B-spline expansion models (Zhang et al. [2006]). Since the control of froth flotation process can be considered as the control of the bubble size distribution and B-spline estimation can identify the PDF accurately with several wights, the control of froth flotation can be transformed to the control of weights in B-spline estimation. Then, the B-spline estimation is chosen to approximate bubble size PDF.

A linear combination of B-spline functions can be used to approximate the PDF of a continuous random variable from a given data. The equation of B-spline estimation $f(x, \alpha)$ of order k is given by

$$f(x, \alpha) = \sum_{i=1}^{n} \alpha_i B_{i,k}(x), t_{\min} \le x \le t_{\max} \qquad (1)$$