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Sensitivity Analysis on Weight and Trajectory Optimization Results for Multistage Guided Missile

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Abstract: Addressed here deals with the problem of weight and trajectory optimization for multi staged anti-air missile. By using the subsystem requirements, such as aerodynamic coefficients, thrust and structure, amount of propellant and weight of each stage can be optimized via appropriate optimization methods. In this paper, the weight and trajectory optimization results are obtained by using the commercial optimization tool, GPOPS-II. To investigate the effect of subsystem parameter and constraints on the optimization result, the sensitivity analysis is conducted.

Keywords: Weight Optimization, Anti-air Guided Missile

1. INTRODUCTION

A guided missile design process includes several iterative steps. Optimized values from previous subsystem are given to the following steps and examined whether they satisfy the other subsystem requirements. During a preliminary design phase, considering subsystem requirements simultaneously can reduce the design iteration numbers and cut off the time and cost required consequently.

Research on trajectory optimization at the initial phase of a missile design is usually conducted to set-up a guideline of the system. The optimization results are used to assess the performance of developed guidance and control system. Weight optimization also can be used as preliminary study on system-level optimization.

Previously, Lee et al. [2015] dealt with trajectory optimization of a long range anti-air missile. The authors solved same trajectory optimization problem via two different methods, GPOPS-II and co-evolutionary augmented Lagrangian method (CEALM). Since GPOPS-II is a tool for optimal control problem and CEALM is a parameter optimization tool, the trajectory optimization problem was transcribed as a parameter optimization problem. Hong et al. [2015] suggested the optimal problem formulation to treat weight and trajectory optimization simultaneously.

In anti-air missiles, analysis on the effect of subsystem parameters is necessary. In addition to this, the angle constraint between the guided missile and a target affects the hit probability and kill probability. Thus, this condition should be included in comparison study. In this paper, the weight and trajectory optimization problem of multi-stage anti-air guided missile is treated. Subsystem requirements, such as a diameter of each stage, a thrust model and aerodynamic coefficients, are also included. With the values given above, optimization results are compared according to the changes in subsystem parameters and constraints.

This paper is organized as follows. The conceptual missile system is introduced and specifications of subsystems are described. Then, the optimal problem for the system is formulated. Also, boundary conditions and constraints are defined. Then, optimization and analysis results are investigated and followed by the summary of this research.

2. ANTI-AIR GUIDED MISSILE

The flight stage configuration of the proposed guided missile is given (Fig. 1). The proposed system is 3-staged missile, of which the first and second stage have propulsion systems and the third stage is a warhead. The missile is controlled by using thrust vectoring, only while the stage 1 and 2 are burning. In the phase 1, stage 1 is launched vertically. Then, the first stage is separated and the missile flies without any control until the second stage ignites. After the stage 2 burns out, it is separated, the warhead flies and intercepts the target.

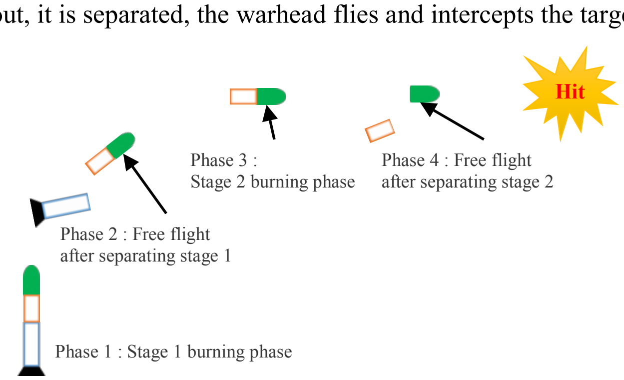

Fig. 1. Flight stage configuration of the proposed guided missile

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The magnitude of thrust force generated by the propulsion system is determined by considering the maximum acceleration at the burn-out moment of each stage. The equation for the thrust force is given as

$$T = m_{b.o.} \times g \times A_{\max} \tag{1}$$

where $m_{b.o.}$ is the burn-out mass of each stage (kg), g is the gravity force (m/s^2), and $A_{\max}$ is the maximum acceleration of the engine represented by g unit (non-dimensional), respectively. The aerodynamic model of guided missile uses a simple model, including induced drag force. The lift and drag force can be obtained as follows.

$$\begin{aligned} C_L &= C_{L_\alpha}\alpha \\ C_D &= C_{D_0} + kC_L^2 \end{aligned} \tag{2}$$

Here, a simple environment model is used. Air density varies according to the altitude, whereas gravity is constant. Detailed values for the missile specification used in this research are summarized in Table 1.

3. PROBLEM FORMULATION

3.1 Optimal Problem Formulation

The equation of motion for weight and trajectory optimization of the missile described in previous section can be written as below.

$$\begin{aligned} \dot{x} &= V_x \\ \dot{z} &= V_z \\ \dot{V}_x &= \left(T\cos\theta - D\cos\gamma - L\sin\gamma\right)/m_{tot} \\ \dot{V}_z &= \left(T\sin\theta - D\sin\gamma + L\cos\gamma\right)/m_{tot} - g \\ \dot{m}_1 &= -m_{p1}/t_{b1} \\ \dot{m}_2 &= -m_{p2}/t_{b2} \end{aligned} \tag{3}$$

where x is downrange, z is altitude, V_x is x -directional velocity, V_z is z -directional velocity, m_1 is stage 1 mass, m_2 is stage 2 mass and m_t is total mass including warhead mass, $m_t = m_1 + m_2 + m_w$. γ is the flight path angle, $\gamma = \tan^{-1}\left(V_z/V_x\right)$ and θ is the pitch attitude angle, $\theta = \gamma + \alpha$. T is thrust, L is lift force, $L = QSC_L$ and D is drag force, $D = QSC_D$ where Q and S denote dynamic pressure and reference area.

Table 1. Missile System Parameter

Variables	Stage 1	Stage 2	Warhead
C_{D_0}		0.4	
C_L	4.5	2.5	1.5
Diameter (m)	0.4	0.4	0.4
I_{sp} (s)	250	250	-
$A_{\max}$ (g)	30	10	-

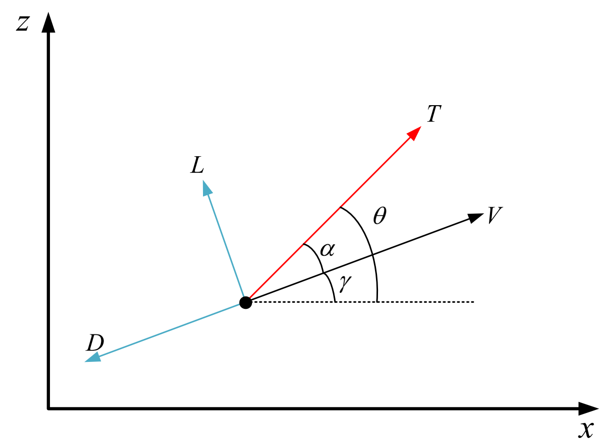

Fig. 2. Missile pitch planar geometry

$m_{p(\cdot)}$ is the propellant mass and $t_{b(\cdot)}$ is the burn-time of each stage, respectively. Missile pitch planar geometry is depicted in Fig. 2.

In this missile, warhead mass m_w is constant as 100kg. Additionally, the 1st stage mass m_1 and 2nd stage mass m_2 are also optimization parameters for weight optimization and free flight time during the phase 2, t_{coast} , is defined as another optimization parameter. The control input for the missile is angle-of-attack (AOA), which will be optimized by the commercial optimization tool, GPOPS-II.

Cost function J for the optimal control problem is defined to minimize the total mass of the missile.

$$\min J = m_t \tag{4}$$

When the second stage is separated, the target should be located in the seeker range. Define d as the distance between the missile and the target at the separation moment and its bound is given below.

$$7km \le d \le 10km \tag{5}$$

3.2 Vertical Launch for GPOPS-II

Since it is difficult to know the approaching direction of a target in advance, the missile should be launched vertically to react to targets from arbitrary directions. Namely, the control input AOA should be zero at the initial time. In the optimization tool used in this research, GPOPS-II, constraints on the control input cannot be designated at the initial and final time. To set the control input zero at the initial time, AOA at the phase 1 is expressed using following equation.

$$\left|\alpha\right| \le U \times k\left(t\right) \tag{6}$$

where $k\left(t\right)$ is a function of time and U is the control input boundary which can be adjusted in GPOPS-II. In phase 1, U is determined as given below.

$$\left|U\right| \le 1 \tag{7}$$