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# Fast Nonsingular Terminal Sliding Mode to Attenuate the Chattering for Missile Interception with Finite Time Convergence

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**Abstract:** This paper proposed a new fast nonsingular terminal sliding mode to design terminal angle constraint guidance for intercepting maneuvering targets with attenuating the chattering of the command in the guidance law. A fast nonsingular terminal sliding mode (FNTSM) is established for dealing with the relationship between Line of sight (LOS) angle and the rate of LOS angle, the disturbance observer (DOB) is set up to calculate the compensation amount for counteracting the time-varying uncertainty produced by the design of guidance and targets' maneuver. Combining with FNTSM and DOB, an adaptive law and a compensated parameter are adopted to ease the coupling relationship between the convergence rate of sliding variable and initial parameters design. Furthermore, the chattering amplitude can be reduced, even can be eliminated in some cases. Theoretical analysis and numerical simulations verify the effectiveness of the proposed method.



*Keywords:* Sliding mode, terminal guidance, missile interception, attenuating the chattering.

## 1. INTRODUCTION

Intercepting a maneuvering target is a challenging issue for a guided missile. The trajectory of a guided missile is typically divided into three segments consisting of lift-off, midcourse, and terminal phase, in these three phases, the terminal homing is a very crucial phase (Das, 2009). To guarantee the guided missile acquire perfect accuracy, terminal homing guidance law designing for guided missile has attracted great attention in the past decades. For increasing the missile's kill probability, improving penetration capabilities, and reducing the warhead size, both terminal angle constraint and near-zero miss distance are vital requirements for missiles' terminal homing guidance law design to successfully intercept an incoming target (Zarchan, 2012; Padhi, 2013).

To satisfy the vital requirements, recent years, terminal angle constraint guidance law based on sliding mode control has become a research hotspot (Zhang, 2012; Wang, 2013; Lu, 2015; Kumar, 2014; Zhou, 2015; He, 2015). Designing sliding surface to guarantee the states to converge in finite time if the sliding variable equals zero, constructing convergence law to make sure the sliding variable converge to zero or in a small region around zero and eliminating uncertainty which influence the convergence quality of the law are the common three parts to propose these sliding mode guidance laws.

Guidance laws in (Zhang, 2012; Lu, 2015) designed the sliding surface based on Nonsingular Terminal Sliding Mode. Guidance laws in (Kumar, 2014; Zhou, 2015) adopted Continuous Nonsingular Terminal Sliding Mode to design the sliding surface. However, these guidance laws can guarantee the states achieve the finite time convergence in the sliding phase. To speed up the convergence rate of states, the guidance law in (He, 2015) recurred to Fast Nonsingular Terminal Sliding Mode (FNTSM) to design the sliding surface.

To design finite time convergence law for the sliding surface, the uncertainty of the system should be suppressed. Guidance laws in (Sun, 2013; Kumar, 2014) were derived based on assuming the upper boundary or the accurate amplitude of uncertainty known, unfortunately, the accurate boundary is hardly known in advance. To solve this problem, guidance laws in (Wang, 2013; Lu, 2015) adopted adaptive scheme to suppress the uncertainty, and the method for proving the quality of convergence has revealed the coupling relationship between the convergence of sliding surface and the uncertainty suppressed. However, the adaptive law will affect the convergence rate of the sliding surface at the same time. (Li, 2015) has pointed out that the initial adaptive parameter in these guidance laws need to be large enough, and will increase as time to a larger value which may cause some instability problems. Obviously, the estimation of uncertainty is crucial for guidance law design, consequently, Extended State Observer (ESO) (Lu, 2015) and General Disturbance Observer (GDOB) (He, 2015) can be applied to estimate the time-varying uncertainty.


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The guidance laws in (He, 2015; Ding, 2013) based on estimator adopted finite-time convergence function to guarantee a quick convergence rate of sliding surface. Unfortunately, the parameters affecting the convergence rate are sensitive to the errors between current states and terminal desired states, and the initial value of parameters may yield large magnitudes (Ding, 2013). Furthermore, the large parameters can affect the precision of the terminal guidance. Even though the sign function in the laws may cause the guidance command chattering with time, and a predefined saturation function utilized instead of sign function may eliminate chattering. Unfortunately, the coupling relationship between the parameters and the initial errors has not been weakened, the chattering phenomenon may emerge and the chattering amplitude may be large enough.

Inspired by the previous work, considering the FNTSM which can combine the relationship between the error and error rate of LOS and easing the coupling relationship among the chattering amplitude, disturbance compensation and convergence rate of sliding variable, a new fast nonsingular terminal sliding mode to attenuate the chattering guidance (SMACG) law is proposed.

## 2. MODES AND DISTURBANCE OBSERVER

### *2.1 Mode Description*

The two dimensions planar terminal homing engagement geometry between a guided missile and the manoeuvring target is depicted in Fig. 1 , where the subscripts M and T denote the guided missile and the manoeuvring target, $\lambda$ and $R$ the LOS angle and the guided missile-manoeuvring target relative range, $V_M$ and $V_T$ the guided missile and manoeuvring target velocity, $\gamma_M$ and $\gamma_T$ the guided missile and manoeuvring target flight path angle, $A_M$ and $A_T$ the guided missile and maneuvring target acceleration, which are normal to their corresponding velocities, respectively.

**Assumption 1**: the guided missile and manoeuvring target are point masses moving in the 2D plane with constant velocities, respectively.

The corresponding equations describing the guided missile-maneuvring target engagement kinematics are formulated as

$$\dot{R} = V_T \cos(\gamma_T - \lambda) - V_M \cos(\gamma_M - \lambda) \tag{1}$$

$$\dot{\lambda} = \frac{1}{R}[V_T \sin(\gamma_T - \lambda) - V_M \sin(\gamma_M - \lambda)] \tag{2}$$

$$\dot{\gamma}_M = \frac{A_M}{V_M} \tag{3}$$

$$\dot{\gamma}_T = \frac{A_T}{V_T} \tag{4}$$

Differentiating (1) and (2) with respect to time yields

$$\ddot{R} = R\dot{\lambda}^2 + A_{TR} - A_M \sin(\lambda - \gamma_M) \tag{5}$$

$$\ddot{\lambda} = -\frac{2\dot{R}\dot{\lambda} + A_{T\lambda} - A_M \cos(\lambda - \gamma_M)}{R} \tag{6}$$

where $A_{TR} \triangleq A_T \sin(\lambda - \gamma_T), A_{T\lambda} \triangleq A_T \cos(\lambda - \gamma_T)$ denote the manoeuvring target acceleration along and normal to the LOS, respectively.

(He, 2015) has analysed the characteristics of the formulation, the differential formulation of relative range is not the main influence factor of the guidance, the main work of designing guidance is concentrated into using $A_M$ to control $\lambda$ for achieving the terminal angle constraint. Therefore, in order to remove the dependence of the guidance law on the term $\cos(\lambda - \gamma_M)$, a new variable is introduced as follows

$$h = A_M + A_{T\lambda} - A_M \cos(\lambda - \gamma_M) \tag{7}$$

Then, the LOS angular rate dynamics (6) can be rewritten as

$$\ddot{\lambda} = -\frac{2\dot{R}\dot{\lambda} - h + A_M}{R} \tag{8}$$

As the unknown parameter of $A_{T\lambda}$, therefore, $h$ should be treated as a time-varying uncertainty.

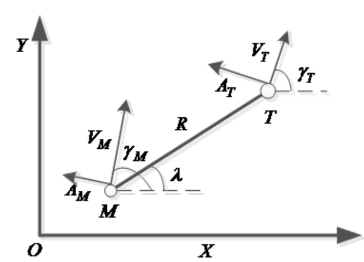


Fig. 1 Terminal homing engagement geometry

**Assumption 3** The lumped uncertainty $h$ is continuous and satisfies

$$\left| d^j h / (dt^j) \right| \le \vartheta \qquad for \quad j = 0,1,2...,r \tag{9}$$

where $\vartheta$ is a positive constant.

**Remark 1**: $h$ covers the lumped uncertainty produced by the manoeuvring target and the guided missile, for example, the ramp manoeuvre, periodic manoeuvre and so on. It may be noted that it is not required to know the bound $\vartheta$ .

### *2.2 Disturbance Observer*

Through some simple manipulations, (8) can be rewritten as

$$d(R\dot{\lambda}) / dt = -\dot{R}\dot{\lambda} + h - A_M \tag{10}$$

Assume $d_1 = R\dot{\lambda}$ and $d_2 = -\dot{R}\dot{\lambda} - A_M$ , a second order DOB for (10) is proposed as follows