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Gain-Scheduled Model-Matching Flight Controller Using Inexact Scheduling Parameters ⋆

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Abstract: This paper addresses the design problem of Gain-Scheduled (GS) flight controller for the lateral-directional motions of a research airplane. The GS flight controller, which is scheduled by an inexactly provided scheduling parameter, *viz.* equivalent air speed measured by Pitot tube, is required to realize model-matching as well as gust suppression under the uncertainties related to onboard actuators. The uncertainty in the provided scheduling parameter is supposed to be represented as neither *pure proportional* nor *pure absolute* uncertainties, but as *mixed* uncertainties of the two types of uncertainties. Two GS flight controllers are designed for our problem; one is designed with the consideration of the uncertainty in the provided scheduling parameter, and the other is designed with the supposition that exact scheduling parameter is given. Flight tests in real environment well illustrate the practicality and the effectiveness of the former GS controller compared to the latter GS controller.

Keywords: LPV systems, gain-scheduled controller, flight controller, mixed uncertainties.

1. INTRODUCTION

It is well-known that Gain-Scheduled (GS) controllers are effective for controlling Linear Parameter-Varying (LPV) systems. Many researchers have therefore tackled the design problem of GS controllers. (A good survey paper written by Hoffmann and Werner (2015) reviews contributions in the literature.) In practical systems, it does not always hold true that exact scheduling parameters are provided to GS controllers due to inevitable uncertainties of measurement equipment, e.g. noise, delay, finite resolutions, etc. In such a case, robustness against the uncertainties in the provided scheduling parameters should be considered for the design of GS controllers, and several researchers have already addressed this issue in Daafouz et al. (2008); Heemels et al. (2010); Jetto and Orsini (2010); Sato and Peaucelle (2013), etc.

Flight test results in Sato (2014) well demonstrate the importance of considering the uncertainties in the provided scheduling parameters. In the paper, model-matching GS flight controller design for the Lateral-Directional (L/D) motions of a research airplane MuPAL-α (Fig. 1) is addressed. Equivalent Air Speed (EAS) is set as the scheduling parameter. Similarly to the most of the usual airplanes, EAS is measured by using Pitot tube. The measurement of EAS suffers a strong effect from its position (so-called, “position error”). In the paper, the uncertainty of the provided EAS is supposed to be represented as *absolute* uncertainty with some additional margins for flight safety, and two GS flight controllers are designed, e.g. nominal and robust GS controllers which are designed, respectively,

Fig. 1. Research airplane MuPAL-α

without and with consideration of the uncertainty in the provided EAS. In flight tests, it is well confirmed that the nominal GS controller does not guarantee the control performance obtained in its design phase. In contrast, it is also well confirmed that the robust GS controller does not show such control performance degradation.

Proportional uncertainties, which differ from the problem setting in Sato (2014), can be used as the representation of the uncertainties in the provided scheduling parameters, as in Daafouz et al. (2008); Sato and Peaucelle (2013).

In practical systems, the uncertainties in the provided scheduling parameters cannot be always represented as *pure absolute* uncertainties or *pure proportional* uncertainties. For example, absolute uncertainty representation is suitable when the values of the scheduling parameters are small, since, in general, noise inevitably exists even when the scheduling parameters are almost zeros; however,

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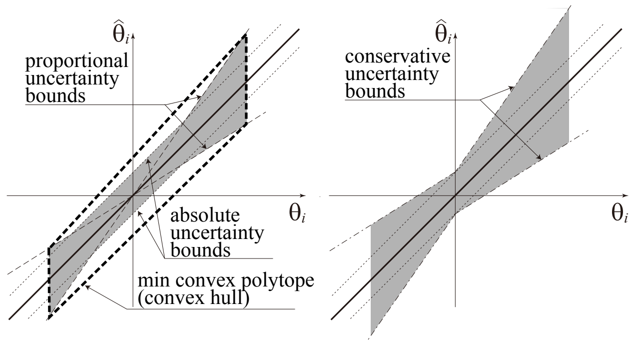

Fig. 2. Illustrative figure for representing provided scheduling parameter with mixed uncertainty (Left: practical situation and minimum convex polytope, i.e. convex hull, right: conservative representation) (Gray region denotes the admissible region of real scheduling parameter θ_i and uncertain scheduling parameter $\hat{\theta}_i$.)

proportional uncertainty representation may be suitable when the values of the scheduling parameters are large, if the resolutions of measurement equipment are low. To address such a problem, *mixed uncertainty* representation has been proposed in Lacerda et al. (2016). That is, the inexactly provided scheduling parameter $\hat{\theta}_i$ for i-th real scheduling parameter θ_i is supposed to be represented as $(\theta_i + \delta_i^a)(1 + \delta_i^p)$ with its absolute uncertainty δ_i^a and its proportional uncertainty δ_i^p. This representation indeed captures the mixed uncertainties; however, it is sometimes too conservative, in particular, as in the case discussed above. To reduce the conservatism, *minimum convex polytope*, i.e. convex hull, which minimally covers the admissible region of the scheduling parameters and the provided ones, is introduced in Sato (2015). In the paper, by exploiting that the admissible region for $(\theta_i, \hat{\theta}_i)$ is given as a convex polytope, the design method in Sato (2011) is directly extended to the mixed uncertainty case, and conservatism reduction is shown for an academic example compared to the representation in Lacerda et al. (2016) when the provided scheduling parameter $\hat{\theta}_i$ lies in the following set:

$$\{\hat{\theta}_i : \theta_i + \underline{\delta}_i^a \le \hat{\theta}_i \le \theta_i + \overline{\delta}_i^a \vee (1 + \underline{\delta}_i^p)\theta_i \le \hat{\theta}_i \le (1 + \overline{\delta}_i^p)\theta_i\}.$$

Here, $\underline{\delta}_i^a, \overline{\delta}_i^a$ ($\underline{\delta}_i^a \le \overline{\delta}_i^a$) and $\underline{\delta}_i^p, \overline{\delta}_i^p$ ($\underline{\delta}_i^p \le \overline{\delta}_i^p$) respectively denote the bounds of absolute and proportional uncertainties for the parameter θ_i. (See Fig. 2.) As shown in Fig. 2, the convex hull contains inadmissible region of $(\theta_i, \hat{\theta}_i)$, i.e. the white region in the bold broken lines (in the left figure); however, the existence region becomes tight compared to the representation given as $(\theta_i + \delta_i^a)(1 + \delta_i^p)$ (in the right figure) and conservatism is consequently reduced.

The objective of this paper is to re-design GS model-matching flight controller, which is robust against the uncertainty in the provided scheduling parameter, *viz.* EAS, for the L/D motions of our research airplane MuPAL-α. The problem setting in this paper has several updates from our previous work in Sato (2014):

- The uncertainty in the provided EAS is supposed to be represented as a mixed uncertainty, not a pure absolute uncertainty, to reduce conservatism of design results.
- Consequently, the admissible true EAS range is slightly enlarged.
- Model-matching as well as gust suppression are simultaneously imposed for GS flight controller design to realize model-matching under gusty conditions.

In GS flight controller design, a design method proposed in Sato (2014), which introduces constant scaling matrices for structured exogenous inputs and performance outputs to the method in Sato (2011) under absolute uncertainties for the provided scheduling parameters, is applied. The reason for the applicability of the method in Sato (2014) to our current problem is as follows: The admissible region of EAS and the provided one is given as a convex polytope, and this property straightforwardly enables us to use the method in Sato (2014) even for the mixed uncertainty case, as long as the admissible region is given as a convex set. For comparison, a nominal GS flight controller, in which exact EAS is supposed to be given, is also designed, and control performances in real environment are examined for the nominal and robust GS flight controllers.

We use the following notations in this paper: $\mathbf{I}_n$ denotes an $n \times n$-dimensional identity matrix, and $\mathbb{R}^n$, $\mathbb{R}^{n \times m}$ and $\mathbb{S}^n$ respectively denote the sets of n-dimensional real vectors, $n \times m$-dimensional real matrices and $n \times n$-dimensional symmetric real matrices. In discrete-time systems, a variable for one step ahead is denoted by the superscript "+".

The notations related to the L/D airplane motions are summarized as follows. v_i [m/s]: inertial sway velocity; v_g [m/s]: sway wind gust; v_a [m/s]: sway airspeed; p [rad/s]: roll rate; ϕ [rad]: roll angle; r [rad/s]: yaw rate; δ_a [rad]: aileron deflection; δ_r [rad]: rudder deflection; δ_{a_c} [rad]: aileron deflection command; δ_{r_c} [rad]: rudder deflection command; V_{true}: true EAS; V_{prov}: provided EAS; and $\delta_{EAS} (:= V_{prov} - V_{true})$: uncertainty in V_{prov}.

The remainder of this paper is as follows: Section 2 defines our addressed problem, Section 3 gives design results and *a posteriori* analysis results, Section 4 shows flight test results, and Section 5 gives concluding remarks.

2. PROBLEM SETTING

In this section, we define our addressed problem. Before showing it, several preliminaries, i.e. modeling of airplane dynamics, etc., are given.

2.1 Modeling

In this subsection, we derive our generalized plant model which is denoted by "$G(V_{true})$" in Fig. 3.

Airplane Dynamics Model The same LPV model in Sato (2014) is used. The LPV model is derived by the following steps: i) Continuous-time LTI models representing the L/D motions of MuPAL-α around wings-level flight condition at 11 equilibrium points in $[51.4, 102.8]$ [m/s] ($[100, 200]$ [kt]) are derived, ii) a continuous-time LPV model is obtained from the 11 LTI models by using least square method, and iii) a discrete-time LPV model is obtained by using Euler method with the onboard computer sampling period ΔT. The state is $[v_i\ p\ \phi\ r]^T$, the