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Zero-Speed Crossing Avoidance with Three Active Reaction Wheels using Set-Point Angular Momentum Management

Ulisses P. Sampaio* Amélie St-Amour Jean de Lafontaine****

*Visiona Tecnologia Espacial S.A., 500 Estrada Dr. Altino Bondesan, Block 02, São José dos Campos, São Paulo, 12.247-023, Brazil (e-mail: ulisses.sampaio@visionaespacial.com.br)
**NGC Aerospace, Ltd, 1650 King Ouest, Bureau 202, Sherbrooke, Québec, J1J 2C3, Canada, (e-mail: ngc@ngcaerospace.com)

Abstract: Reaction wheels (RW) are common actuators for attitude control of three-axis stabilized satellites. By exchanging momentum with the spacecraft's body, a set of three non-planar RW is able to ensure the off-nadir fine pointing capabilities of agile high-resolution imaging satellites. Nonetheless, due to stiction, RW can cause considerable attitude jitter when their angular velocity is close to zero. This paper proposes a method to analyze the maximum off-nadir pointing angles that are allowable without causing zero-speed crossing for a spacecraft using only three active RW. In sequence, by applying this method for a sun-synchronous mission, it is shown that angular momentum management can be used to give the satellite an adequate roll slewing margin by placing the reaction wheel momentum set point in an appropriate position. This method can be applied to any mission that relies on three active RW and requires high pointing stability.

Keywords: Spacecraft dynamics, control, angular momentum, reaction wheel.

1. INTRODUCTION

Most current high-resolution imaging satellites require high attitude pointing stability during imaging and commonly rely on reaction wheels (RW) for attitude control. Since stiction can cause considerable attitude jitter when RWs have near-zero angular velocity, it is important to avoid that any active reaction wheel crosses zero while imaging.

Different methods were developed in the past for RW zero-speed crossing avoidance. Kron et al, for instance, performs zero crossing avoidance by actively distributing angular momentum between four active RW. Using only three active wheels to reduce power consumption, on the other hand, the PROBA-V satellite chooses the positon of the total spacecraft angular momentum ($\vec{h}_{TOT}$) to ensure that, while nadir looking, zero-speed crossing only occurs near the North and South poles (de Lafontaine et al).

This paper presents a mathematical analysis of the maximum roll angle that is allowable without causing RW zero-speed crossing for an Earth-observation spacecraft using three active wheels. Using its results, a method is then proposed to determine an adequate inertial position for $\vec{h}_{TOT}$ in order to avoid zero-speed crossing while acquiring off-nadir images over a specified region of interest.

Finally, this method is applied to reference imaging satellite at a 721 km altitude Sun-Synchronous orbit that is designed to take images with 3-day revisit between latitudes $\pm 60\ deg$.

2. ASSUMPTIONS AND DEFINITIONS

2.1 Total Angular Momentum

In an unperturbed orbit, the total satellite angular momentum ($\vec{h}_{TOT}$) is inertially fixed. This angular momentum is the sum of the spacecraft's body angular momentum and the angular momentum stored in the RW. As long as there are three non-planar active RW, a spacecraft rotation around any direction can be performed by exchanging momentum between the spacecraft's body and the RW (while conserving the system's total angular momentum). This can be observed in the following equation:

$$\vec{h}_{TOT} = \vec{\vec{J}} \cdot \vec{\omega}_B + \textstyle\sum_i \left(J_{s_i} \omega_{rel_i}\right) \vec{a}_i \qquad (1)$$

with the $\vec{\vec{J}}$ inertia given by:

$$\vec{\vec{J}} = \vec{\vec{J}}_B + \textstyle\sum_i \left(J_{t_i} \left(\vec{\vec{1}} - \vec{a}_i \vec{a}_i \right) + J_{s_i} (\vec{a}_i \vec{a}_i) \right) \qquad (2)$$

where $J_{t_i}, J_{s_i}, \omega_{rel_i}$ and $\vec{a}_i$ are, respectively, the transverse inertia, the principal axis inertia, relative angular velocity and principal axis vector of the i-th RW, $\vec{\vec{J}}_B$ is the spacecraft body inertia without the RW and $\vec{\omega}_B$ is the spacecraft body angular rate.

From (1), it can be seen that, for a constant $\vec{h}_{TOT}$, a change in $\vec{\omega}_B$ is obtained by changing the RWs' relative angular velocities.

The following sections assume that three active reaction wheels are used for attitude control.

2.2 Reaction Wheel Angular Momentum Set Point

For a satellite with no orbital perturbations that performs imaging scan with constant angular velocity with respect to its orbit (ex: push broom scan), $\vec{\omega}_B$ will be inertially fixed while imaging.

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If there are no orbital perturbations, $\vec{h}_{TOT}$ is also constant. Thus, we have the following relation:

$$\sum_i (J_{s_i} \omega_{rel_i}) \vec{a}_i = \vec{h}_{TOT} - \vec{J} \cdot \vec{\omega}_B \equiv \vec{S}_p \tag{3}$$

where $\vec{S}_p$ is defined as the inertially-fixed RW angular momentum set point.

Since each RW's principal axis $\vec{a}_i$ is rotating with the spacecraft (and thus is not inertially fixed), for $\vec{S}_p$ to be constant, ω_{rel_i} must also be varying over the orbit (assuming three active wheels with non-zero speed).

Zero-speed crossing occurs when the set point vector is in the plane formed by two reaction wheels. The relative speed ω_{rel_k} of the third wheel must then be zero. This is mathematically represented by the following equation:

$$\vec{S}_p \cdot \left(\vec{a}_i \times \vec{a}_j / \left\| \vec{a}_i \times \vec{a}_j \right\| \right) = 0 \Longrightarrow \omega_{rel_k} = 0, i \neq j \neq k \tag{4}$$

where $\vec{a}_i \times \vec{a}_j$ is a vector pointing along the normal of the plane defined by RWs i and j.

Only torques produced by external forces can alter the satellite's $\vec{h}_{TOT}$ vector. For instance, active use of magnetorquers, propulsion, differential drag or solar pressure can place $\vec{h}_{TOT}$ vector close to a desired inertially fixed point. Thus, for a fixed $\vec{\omega}_B$, it is possible to place $\vec{S}_p$ in the position of choice by controlling $\vec{h}_{TOT}$.

From (4), it can be seen that zero crossing avoidance for three active RW consists in finding a convenient set point $\vec{S}_p$.

2.3 Consideration of the Angular Momentum Sphere

In practice, due to the low precision of the external force actuators and due to the presence of small environmental attitude perturbation torques (ex: gravity gradient, residual magnetic moment), the angular momentum set point is never actually exactly equal to the desired vector ($\vec{S}_p \neq \vec{S}_{p,desired}$).

Nonetheless, by monitoring $\vec{h}_{TOT}$, it is feasible to always keep $\vec{S}_p$ inside an angular momentum sphere around the desired set point vector $\vec{S}_{p,desired}$. Let R_s be the radius of the angular momentum sphere. Zero crossing will be avoided as long as the absolute distance d between the sepoint $\vec{S}_p$ and any plane formed by two RWs is larger than R_s:

$$\vec{S}_p \cdot \left(\vec{a}_i \times \vec{a}_j / \left\| \vec{a}_i \times \vec{a}_j \right\| \right) = d > R_s \tag{5}$$

3. GENERAL FORMULATION

3.1 Modified Inertial Frame

For convenience, a new inertial frame $\mathcal{F}_{I^*}$ with unit vectors $\vec{\mathcal{F}}_{I^*} = \left[\vec{I}_x^* \quad \vec{I}_y^* \quad \vec{I}_z^* \right]^T$ is defined as being coincident with the spacecraft's roll-pitch-yaw frame $\mathcal{F}_Q$, with unit vectors $\vec{\mathcal{F}}_Q = \left[\vec{Q}_x \quad \vec{Q}_y \quad \vec{Q}_z \right]^T$, when the spacecraft is at the ascending node (argument of latitude u equals zero).

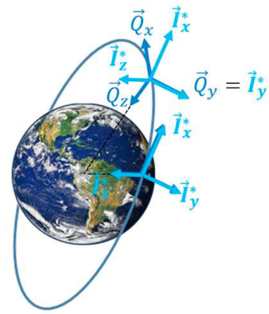

Fig. 1. $\mathcal{F}_{I^*}$ and $\mathcal{F}_Q$ frames.

3.2 Roll margin

A generic set point vector can be represented in component form in $\mathcal{F}_{I^*}$ frame as:

$$\vec{\mathcal{F}}_{I^*} \cdot \vec{S}_p \equiv \underline{S}_p^{I^*} = S_p \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} \tag{6}$$

where $[s_1 \quad s_2 \quad s_3]^T$ are the components of a unit vector and $S_p = \left\| \vec{S}_p \right\|$.

The normal direction to the plane defined by wheels i and j can be represented in component form in body frame $\mathcal{F}_B$ as the following unitary vectors:

$$\begin{aligned} \vec{\mathcal{F}}_B \cdot \vec{n}_{ij,+} &\equiv \underline{n}_{ij,+}^B = \frac{(\underline{a}_i^B)^\times \underline{a}_j^B}{\left\| (\underline{a}_i^B)^\times \underline{a}_j^B \right\|} = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}, i \neq j \\ \vec{\mathcal{F}}_B \cdot \vec{n}_{ij,-} &\equiv \underline{n}_{ij,-}^B = -\frac{(\underline{a}_i^B)^\times \underline{a}_j^B}{\left\| (\underline{a}_i^B)^\times \underline{a}_j^B \right\|} = \begin{bmatrix} -n_1 \\ -n_2 \\ -n_3 \end{bmatrix}, i \neq j \end{aligned} \tag{7}$$

where $\vec{n}_{ij,+}$ is the normal in a positive direction (according to some convention) and $\vec{n}_{ij,-}$ is in the opposite (negative) direction.

Representing the equality of (5) in $\mathcal{F}_B$ frame:

$$\left(\underline{S}_p^B \right)^T \underline{n}_{ij,+}^B = d_+ \tag{8}$$

with a similar equation for d_-using $\vec{S}_p \cdot \vec{n}_{ij,-}$.

It can also be shown that:

$$\underline{S}_p^B = \underline{C}_{B,I^*} \underline{S}_p^{I^*} = \underline{C}_1(\phi) \underline{C}_2(\theta - u) S_p \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} \tag{9}$$

where $\underline{C}_{B,I^*}$ is the rotation matrix from $\mathcal{F}_{I^*}$ to $\mathcal{F}_B$ frame, $\underline{C}_1$ and $\underline{C}_2$ are, respectively, first and second axis rotations and $\phi, \theta,\ u$ are the spacecraft's current roll, pitch and argument of latitude angles.

By developing (8) and using (9), it can be easily demonstrated that:

$$A_+ \cos\phi - B_+ \sin\phi + D_+ = \frac{d_+}{S_p} \tag{10}$$