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Angular Momentum Based Steering Approach for Control Moment Gyroscopes ⋆

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Abstract: With increasing requirements on agile satellite missions, control moment gyroscopes (CMGs) become the actuator of choice due to their high torque amplification capability. In order to handle their inherent singularity issues, many steering laws have been developed that map a desired torque profile to a commanded gimbal rate profile. As CMGs are internal momentum exchange devices, alternative steering laws exist in the angular momentum/gimbal angle domain. In this paper, the two steering approaches are analyzed conceptually and their performance regarding the handling of singularities is evaluated in simulations. For a CMG array in roof configuration, the angular momentum/gimbal angle steering approach proves to be a viable choice for attitude control tasks.

Keywords: Control moment gyroscopes, satellite control, steering laws

1. INTRODUCTION

With their high torque amplification capability, single-gimbal control moment gyroscopes (CMGs) are the actuator of choice for missions with high torque requirements such as large space structures or agile satellites. The main hazard of single-gimbal CMG arrays are singular states, where the controllability of the spacecraft is lost around one or even two axes. Extensive analyses of singularities have been performed by Margulies and Aubrun (1978), Kurokawa (1998), and Wie (2004). For attitude control tasks, much research has therefore been conducted in the development of steering laws that generate appropriate gimbal rate profiles in order to follow commanded torque profiles and avoid singularities (for a comprehensive overview, see Kurokawa, 2007).

As CMGs are internal momentum exchange devices, a commanding based on desired angular momentum states and, hence, gimbal angles is a feasible alternative to the traditional torque/gimbal rate based approach and has been treated in some publications (Yoshikawa, 1974; Verbin and Lappas, 2013). In this paper, the two steering approaches are applied to a 4-CMG array in roof configuration with the focus being on their respective performance near and at singularities. The dynamics of the CMG array are introduced in section 2, while a general description of the steering approaches is provided in section 3. Finally, section 4 comprises a conceptual analysis of the different steering principles, followed by simulation results.

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2. CMG ARRAY IN ROOF CONFIGURATION

A constant-speed, single-gimbal CMG as shown in Fig. 1 is

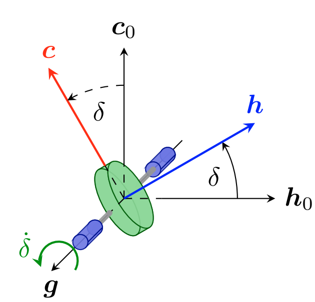

Fig. 1. Single-gimbal CMG

a spinning flywheel mounted such that it can rotate around a gimbal axis, $\boldsymbol{g}$, perpendicular to its spin axis. As a result, the absolute value of its angular momentum $\boldsymbol{h}$ is constant, whereas its direction changes continuously along a circle. Due to gyroscopic effects, the rotation of the gimbal causes a torque perpendicular to both the angular momentum and the gimbal axis. The output torque direction is then defined as

$$\boldsymbol{c}(\delta) = \boldsymbol{g} \times \boldsymbol{h}(\delta), \quad (1)$$

where δ is the *gimbal angle*, i.e., the angle between an arbitrarily defined origin of the gimbal motion and its current orientation.

Figure 2 shows a CMG array in roof configuration consisting of four CMGs. Let $\{\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}\}$ be a Cartesian coordinate frame fixed to the spacecraft body. Then the two angular momenta $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$—and therefore also $\boldsymbol{c}_1$ and $\boldsymbol{c}_2$—are confined to the xy-plane. Analogously, $\boldsymbol{h}_3, \boldsymbol{h}_4$, and their

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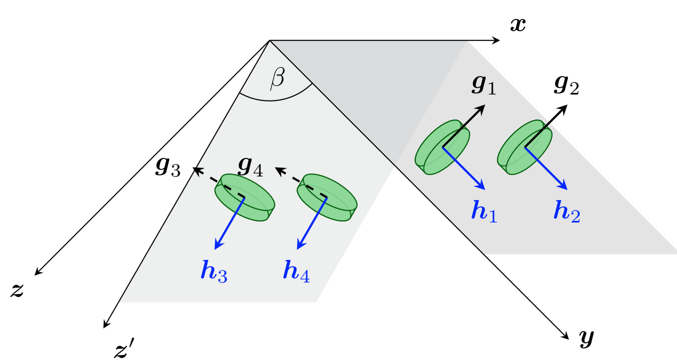

Fig. 2. 4-CMG array in roof configuration

derivatives are confined to the xz'-plane. The skew angle β is a design variable with which the realizable angular momentum (and hence, torque) envelope of the CMG array can be shaped according to mission requirements. In the following, assume that all CMGs have the same angular momentum $|\boldsymbol{h}_i| = h$ and let $\beta = \pi/2$ (hence, $\boldsymbol{z}' = \boldsymbol{z}$). The total angular momentum of the CMG array is then

$$\boldsymbol{H}(\boldsymbol{\delta}) = \sum_{k=1}^{4} \boldsymbol{h}_k(\delta_k) = h \begin{bmatrix} s.\delta_1 + s.\delta_2 - s.\delta_3 - s.\delta_4 \\ c.\delta_1 + c.\delta_2 \\ c.\delta_3 + c.\delta_4 \end{bmatrix}, \quad (2)$$

where $s.\delta_i \equiv \sin(\delta_i)$ and $c.\delta_i \equiv \cos(\delta_i)$. The output torque of the CMG array is obtained by differentiating (2) w.r.t. time:

$$\boldsymbol{T}(\boldsymbol{\delta}, \dot{\boldsymbol{\delta}}) = \frac{\mathrm{d}\boldsymbol{H}(\boldsymbol{\delta})}{\mathrm{d}t} = \frac{\partial \boldsymbol{H}}{\partial \boldsymbol{\delta}} \frac{\partial \boldsymbol{\delta}}{\partial t} \equiv \boldsymbol{C}(\boldsymbol{\delta})\, \dot{\boldsymbol{\delta}} \quad (3)$$

In (3), $\boldsymbol{C}(\boldsymbol{\delta})$ is the Jacobian of the CMG array:

$$\boldsymbol{C}(\boldsymbol{\delta}) = [\boldsymbol{c}_1 \ \cdots \ \boldsymbol{c}_4] = h \begin{bmatrix} c.\delta_1 & c.\delta_2 & -c.\delta_3 & -c.\delta_4 \\ -s.\delta_1 & -s.\delta_2 & 0 & 0 \\ 0 & 0 & -s.\delta_3 & -s.\delta_4 \end{bmatrix}$$

An important property of CMG arrays is the existence of *singularities*, i.e., configurations where the Jacobian loses rank. In that case, there exists a singular direction, $\boldsymbol{u}_\mathrm{s}$, in which no output torque can be generated as all CMGs' output torques are orthogonal to said direction:

$$\boldsymbol{c}_k^\mathrm{T} \boldsymbol{u}_\mathrm{s} = 0 \qquad \forall k$$

Note that CMG singularities are a physical phenomenon and hence cannot be mitigated by choosing different mathematical descriptions. For attitude control applications, singularities pose a crucial problem as they may cause a loss of controllability of the spacecraft. As a result, the proper commanding of the gimbals is an important part of any CMG-based attitude control system.

3. STEERING APPROACHES FOR CMG ROOF ARRAY

3.1 Torque/Gimbal Rate Steering Laws

In the traditional attitude control architecture, a feedback controller—possibly in combination with some feedforward guidance profile—provides a torque command that has to be realized by the spacecraft's actuators. In case of CMGs, the commanding task consists of finding a gimbal rate profile such that the CMG array's output torque follows the desired torque profile, i.e., of inverting (3). At this point, the proper handling of singularities becomes relevant as when $\boldsymbol{C}(\boldsymbol{\delta})$ comes close to losing rank, the gimbal rates necessary to provide a specific torque increase rapidly beyond hardware limitations. On the other hand, with four CMGs to produce a three-dimensional torque, there is one additional degree of freedom for the gimbal motion. This can be exploited to mitigate the effects of singularities, e.g., by adding a null motion that does not generate any torque, yet moves the array away from singular configurations. A survey over various steering laws can be found in Kurokawa (2007); in this paper, the so-called generalized singularity-robust (GSR) inverse (Wie et al., 2001) is chosen:

$$\boldsymbol{C}^\dagger = \boldsymbol{C}^\mathrm{T} \left(\boldsymbol{C}\boldsymbol{C}^\mathrm{T} + \lambda \boldsymbol{E} \right)^{-1} \quad (4)$$

with:

$$\lambda = 0.01 \exp\left(-10 \det(\boldsymbol{C}\boldsymbol{C}^\mathrm{T}) \right)$$

$$\boldsymbol{E} = \begin{bmatrix} 1 & \epsilon_3 & \epsilon_2 \\ \epsilon_3 & 1 & \epsilon_1 \\ \epsilon_2 & \epsilon_1 & 1 \end{bmatrix}$$

$$\epsilon_1(t) = 0.01 \sin(0.5\,\pi\, t)$$
$$\epsilon_2(t) = 0.01 \sin(0.5\,\pi\, t + \pi/2)$$
$$\epsilon_3(t) = 0.01 \sin(0.5\,\pi\, t + \pi)$$

3.2 Angular Momentum/Gimbal Angle Steering Laws

As CMGs are internal momentum exchange devices, an attitude controller may also provide a desired angular momentum profile of the spacecraft, which in turn directly defines the desired angular momentum of the CMG array due to the conservation of angular momentum. Formally, the commanding task is analogous to section 3.1, i.e., find a gimbal angle profile such that the CMG array's angular momentum follows the desired profile by inverting (2). Even though (2) is nonlinear in the gimbal angles, the specific geometry of the roof array allows for an intuitive solution by separating the angular momentum contributions of the two CMG pairs. As is clear from Fig. 2, angular momentum around the y-axis (z-axis) can only be provided by the xy-pair (xz-pair). Therefore, the one degree of freedom pertains to the allocation of the desired x-axis angular momentum to the pairs, for which different approaches have been proposed by Yoshikawa (1974) and Verbin and Lappas (2013). In this paper, the proportional allocation of the former is chosen. Let $\boldsymbol{U} = [U_x \ U_y \ U_z]^\mathrm{T}$ be the total desired angular momentum and denote the desired angular momentum of the two pairs by $\boldsymbol{h}_{xy}$ and $\boldsymbol{h}_{xz}$. Then:

$$\boldsymbol{h}_{xy} = \begin{bmatrix} X_1 \\ U_y \\ 0 \end{bmatrix} \qquad \boldsymbol{h}_{xz} = \begin{bmatrix} X_2 \\ 0 \\ U_z \end{bmatrix}$$
$$X_1 = \frac{x_1}{x_1 + x_2} U_x \qquad X_2 = \frac{x_2}{x_1 + x_2} U_x \quad (5)$$
$$x_1 = \sqrt{(2h)^2 - U_y^2} \qquad x_2 = \sqrt{(2h)^2 - U_z^2}$$

Within each pair, the desired angular momentum directly defines its gimbal angles, e.g. for the xy-pair:

$$\delta_{a,b} = \operatorname{atan2}(h_{xy,x}, h_{xy,y}) \pm \arccos\left(|\boldsymbol{h}_{xy}|/(2h)\right) \quad (6)$$